Topic 3:
FRACTIONS, PERCENTAGES AND DECIMALS

Key Words

recurring, numerator, denominator, terminating, non-terminating, reciprocal, whole

By the end of this topic, you should be able to:

i) describe different types of fractions.
ii) convert improper fractions to mixed numbers and vice versa.
iii) work out problems from real-life situations.
iv) add, subtract, divide and multiply decimals.
v) convert fractions to decimals and vice versa.
vi) identify and classify decimals as terminating, non-terminating and recurring decimals.

vii) convert recurring decimals into fractions.
viii) convert fractions and decimals into percentages and vice versa.
ix) calculate a percentage of a given quantity.
x) work out real-life problems involving percentages.

Introduction

In Chapter Two you studied place values in number bases. In this topic, you will use knowledge of place values to manipulate fractions, decimals and percentages. You will convert fractions to decimals, decimals to percentages and vice versa.

Sub-topic 3.1: Describe Different Types of Fractions

Activity 3.1

Create a park of different cards and label them with different types of fractions, decimals and percentages.

From the park of the cards, you pick a card and place it in the most appropriate play area.

Observe the fractions in each play area by looking at the denominators and numerators.

In your groups explore and explain the common of the classification made in the different play areas.

Exercise

1. Sarah shades 3/7 of a shape. What fraction of the shape is left unshaded?
2. A cake is divided into 12 equal parts. John eats 3/12 of the cake and Kate eats another 1/12. What fraction of the cake is left?
3. A car park contains 20 spaces. There are 17 cars parked in the car park.

a) What fraction of the car park is full?
b) What fraction of the car park is empty?

4. Ali eats 3/10 of the sweets in a packet.
Tariq eats another 4/10 of the sweets.

a) What fraction of the sweets has been eaten?
b) What fraction of the sweets is left?

5.
a) Draw a square with its four lines of symmetry.
b) Shade 3/8 of the shape.
c) Shade another 2/8 of the shape.
d) What is the total fraction now shaded?
e) How much is left unshaded?

Sub-topic 3.2: Convert Improper Fractions to Mixed Numbers and Vice Versa

Mixed Numbers and improper Fractions

So far you have worked with fractions of the form a/b where a < b, e.g.

¾, 2/7, 5/6 …

You also need to work with what are sometimes called *improper* fractions, e.g. 5/4, 7/2, which are of the form a/b when **a** and **b** are whole numbers and **a > b.**

Example

Convert 13/4 into an improper fraction.

Solution

13 ÷ 4 = 3 remainder 1

This is written as 3 ¼.

Exercise

1. Draw diagrams to show these improper fractions:
(a) 7/2 (b) 8/3 (c) 18/5

Write each improper fraction as a mixed number.

2. Convert these mixed numbers to improper fractions.
(a) **1** 3/5 (b**) 7** 1/3 (c) **3** 4/5 (d) **6** 1/9

3. Write these fractions in order of increasing size.
6 ½ , 18/5 , 3 ¼ , 5 1/3 , 17/3

4. In an office there are 2 ½ packets of paper. There are 500 sheets of paper in each full packet. How many sheets of paper are there in the office?
5. A young child is 44 months old. Find the age of the baby in years as a mixed number in the simplest form.

Sub-topic 3.3: Operations on Fractions

In the previous sub-topic, you studied how to find equivalent fractions. In this sub-topic you are going to use the knowledge of equivalent fractions to add and subtract fractions.

3.3.1: Work out problems from real-life situations

Now we start to use fractions in a practical way.

Example

(a) Find 1/5 of UGX. 10000

(b) Find 4/5 of UGX. 100,000

You can, do this practically, but it is much easier to work out.

(a) 1/5 of 10000 = 1/5 x 10000 = 2000

(b) 4/5 of 100000 = 4/5 x 100000 = 400000/5 = 80,000

Exercise

1. Find:

(a) ½ of 12 (b) 1/8 of 40 (c) ¼ of 32

2. Find:

(a) 2/9 of 18 (b) 7/9 of 45 (c) 7/8 of 56

3. In a test, there are 30 marks. Nasim gets 3/5 of the marks. How many marks does she get?

4. In a certain school there are 550 pupils. If 3/50 of the pupils are left-handed, how many left-handed pupils are there in the school?

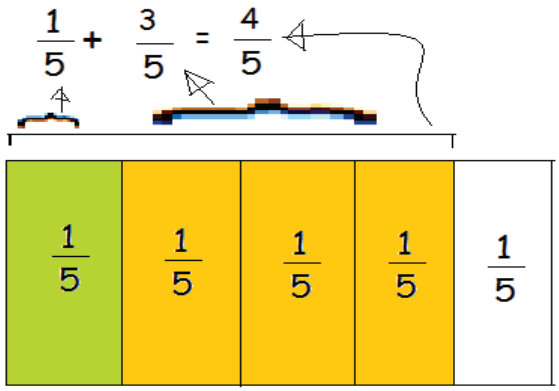
Activity 3.3: Addition of Fractions

In your groups, use a sheet of paper to work out $\frac{1}{5}+\frac{3}{5}$. Fold the paper into five equal parts shade off one part of the five equal parts

Shade the three parts of the five equal parts

How many parts have been shaded?

Represent the shaded parts in a fraction form. Show the working.



Activity 3.4: Addition of Fractions with the Same Denominators

Slice a hexagon into 6 pieces:

Each piece  is $\frac{1}{6}$ of the hexagon. Right?

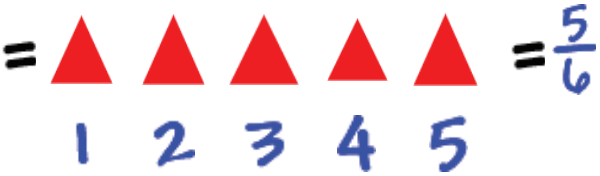
And  is $\frac{4}{6}$ of the hexagon.

So, what if we wanted to add

$$\frac{1}{6} + \frac{4}{6} ?$$

Hmm... that would be

Count them up



So $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

In your groups, use the same method to work out the following:

a) $\frac{3}{7}+\frac{2}{7}$

b) $\frac{5}{9}+\frac{4}{9}$

3.3.2: Adding Fractions with the different Denominators

In the previous topic you studied about lowest common multiple. In this section, you will apply the knowledge of LCM.

$\frac{1}{2}+\frac{1}{3}$

Change the $\frac{1}{2}$ using the knowledge of equivalent fractions

$\frac{1\times3}{2\times3}=\frac{3}{6}$

Change the $\frac{1}{3}$ using the knowledge of equivalent fractions

So $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$

$\frac{3+2}{6}=\frac{5}{6}$

The main rule of this game is that we cannot add the fractions until the denominators are the same!

We need to find something called the least common denominator (LCD)..which is the LCM of our denominators, 2 and 3.

The LCM of 2 and 3 is 6. So, our LCD 6.

We need to make this our new denominator

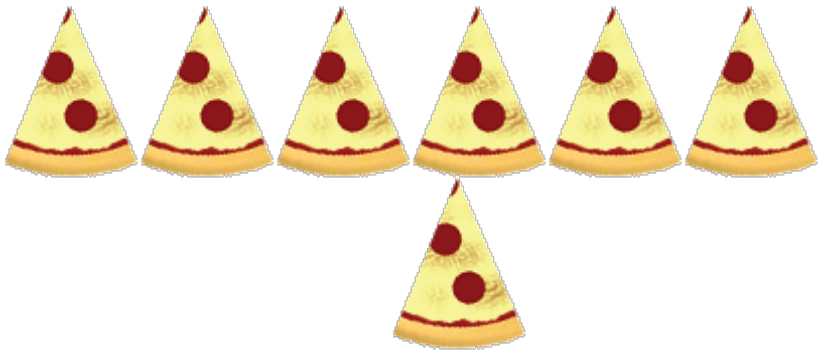
3.3.3: Subtraction of Fractions with Same Denominators

Let's try $\frac{7}{8} - \frac{3}{8}$

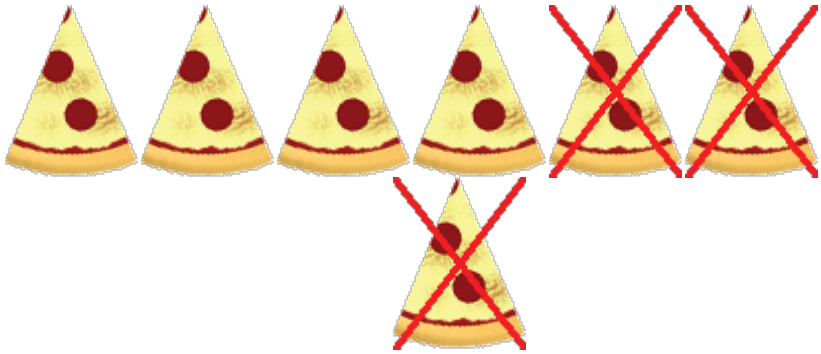
Look at a Chapatti in a conical shape cut into 8 pieces. Each piece

 is $\frac{1}{8}$ of the Chapatti.

$\frac{7}{8}$



Take $\frac{3}{8}$ away (that's 3 pieces):



We're left with 4 pieces - that's.

So $\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$

But, look what we really did!

We just subtracted the numerators!

$\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8}$ which is $\frac{1}{2}$

3.3.4: Subtraction of Fractions with Different Denominators

Subtraction works the same way.

$$\frac{6}{11} - \frac{3}{22}$$

The LCM of 11 and 22 is 22... So, the LCD is 22.

We just need to change the $\frac{6}{11}$.

$$\frac{6 \times 2}{11 \times 2} = \frac{12}{22}$$

$$\text{So } \frac{6}{11} - \frac{3}{22} = \frac{12}{22} - \frac{3}{22} = \frac{12-3}{22} = \frac{9}{22}$$

Done!

3.3.5: Addition of Mixed Fractions

What if we need to add

$$3 + \frac{7}{8}?$$

Hey, remember, that's just $3\frac{7}{8}$.

Done!

That was easy, but, what about mixed numbers?

How about this?

$$3\frac{2}{5} + 1\frac{4}{7}$$

All we have to do is change these to improper fractions... Then we can add them!

$$3\frac{2}{5} + 1\frac{4}{7} = \frac{17}{5} + \frac{11}{7}$$

change to improper fractions

$$= \frac{17 \times 7}{5 \times 7} + \frac{11 \times 5}{7 \times 5} = \frac{119}{35} + \frac{55}{35}$$

change to the LCD of 35

$$= \frac{119 + 55}{35} = \frac{174}{35}$$

3.3.6: Subtraction of Mixed Fractions

$$5 - \frac{3}{8}?$$

Well, we can't just stick it together like we would if it was addition.

We need to get a common denominator... But, the 5 does not even have a denominator!

That's OK... Just think of a Chapatti cut into 8 pieces...

How many pieces would there be in 5 chapattis? Yep!

$$5 \times 8 = 40 \text{ pieces}$$

So $5 = \frac{40}{8}$

Check it: $\frac{40}{8}$ is the same as $40 \div 8$ which is $= 5$. Yep!

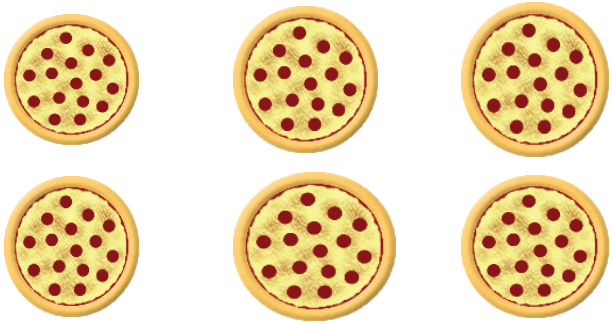
Back to the problem:

$$5 - \frac{3}{8} = \frac{40}{8} - \frac{3}{8} = \frac{40-3}{8} = \frac{37}{8} = 4\frac{5}{8}$$

What's $\frac{1}{3} \times 6$?

Well, that's $\frac{1}{3}$ of 6. Think about it:

You have 6 chapattis.



and you get to eat $\frac{1}{3}$ of them.

This is like splitting up the chapatti between 3 people:

 You get 2 chapattis	 Your friend gets 2 chapattis	 And your dog gets 2 chapattis
 You get 2 chapattis	 Your friend gets 2 chapattis	 And your dog gets 2 chapattis

So $\frac{1}{3}$ of 6 is 2.

But, how do we do this with just math? EASY!

We know how to multiply two fractions... Right?

So, just make both things be fractions. Check it out:

$\frac{1}{3} \times 6$

$\frac{1}{3}$ is already a fraction...

But, what about the 6?

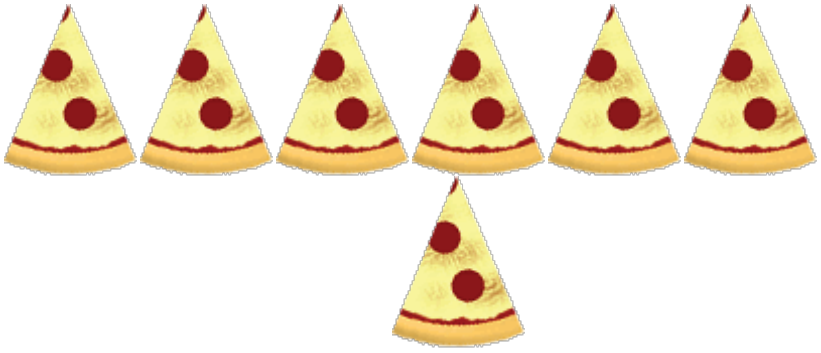
Guess what? We can write 6 as $\frac{6}{1}$.

Let's try

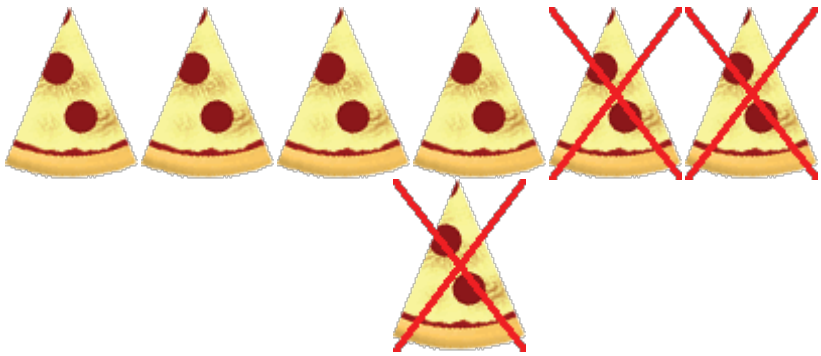
$$\frac{7}{8} - \frac{3}{8}$$

Look at a chapatti cut into 8 pieces. Each piece  is $\frac{1}{8}$ of the Chapatti.

$\frac{7}{8}$



Take $\frac{3}{8}$ away (that's 3 pieces):



We're left with 4 pieces, that's.

So $\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$

But, look at what we really did!

We just subtracted the numerators!

$\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8}$ which is $\frac{1}{2}$

3.3.7: Subtraction of Fractions with Different Denominators

Subtraction works the same way.

$\frac{6}{11} - \frac{3}{22}$

The LCM of 11 and 22 is 22... So the LCD is 22.

We just need to change the $\frac{6}{11}$.

$$\frac{6 \times 2}{11 \times 2} = \frac{12}{22}$$

$$\text{So } \frac{6}{11} - \frac{3}{22} = \frac{12}{22} - \frac{3}{22} = \frac{12-3}{22} = \frac{9}{22}$$

Done!

3.3.8: Multiplication of Fractions

What's $\frac{1}{3} \times 6$?

Well, that's $\frac{1}{3}$ of 6. Think about it...

You have 6 chapattis.

and you eat $\frac{1}{3}$ of them.

This is like splitting up the pizza between 3 people:

 You get 2 chapattis	 Your friend gets 2 chapattis	 And your dog gets 2 chapattis

So $\frac{1}{3}$ of 6 is 2.

But, how do we do this with just math? EASY!

We know how to multiply two fractions... Right?

So, just make both things be fractions. Check it out:

$\frac{1}{3} \times 6$

$\frac{1}{3}$ is already a fraction...

But, what about the 6?

Guess what? We can write 6 as $\frac{6}{1}$.

Think about it:

$\frac{6}{1}$ is the same as $6 \div 1$... which is 6!

(You can do this with any number!)

Back to the problem:

$$\frac{1}{3} \times 6 = \frac{1}{3} \times \frac{6}{1} = \frac{1 \times 6}{3 \times 1} = \frac{6}{3} = 2$$

Just what we figured!

3.3.9: Multiplying Mixed Fractions

What about this?

$$2\frac{3}{5} \times 3\frac{1}{7}$$

Yikes! I am sure I don't want to try to think about pizza for this one!

Let's stick to the math:

Again, let's change these into improper fractions and go for it!

$$2\frac{3}{5} \times 3\frac{1}{7} = \frac{13}{5} \times \frac{22}{7} = \frac{286}{35} = 8\frac{6}{35}$$

This is super easy!

Let's just do one:

$$\frac{1}{3} \times \frac{9}{10}$$

We just multiply straight across...

$$\frac{1}{3} \times \frac{9}{10} = \frac{1 \times 9}{3 \times 10} = \frac{9}{30} = \frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

Then just reduce it

Now, think about it...

$$\frac{1}{3} \times \frac{9}{10} \quad \frac{1}{3} \text{ of } \frac{9}{10}$$

Cut a pizza into 10 pieces like

and look at 9 of the pieces:

We want $\frac{1}{3}$ of these $\left(\frac{1}{3} \text{ of } \frac{9}{10}\right)$

That would be 3 pieces. Right?

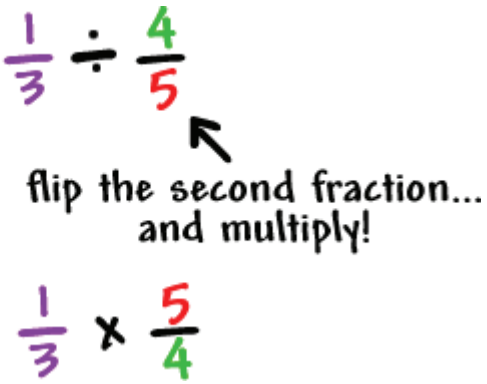
That's $\frac{3}{10}$!

Doing math is coooooool!

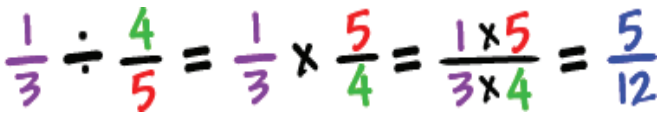
Now that we understand what to do, we can just go for it.

3.3.10: Division of Mixed Fractions Flip And Multiply

Check it out:

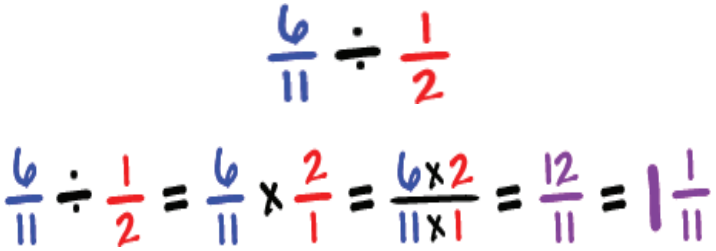


That's it -- then GO FOR IT!



Done!

Look at another one:



Use the same trick you do when multiplying by changing everything to fractions and then go for it!

Check it out:

$$\frac{9}{17} \div 3$$

$$\frac{9}{17} \div 3 = \frac{9}{17} \div \frac{3}{1} = \frac{9}{17} \times \frac{1}{3} = \frac{9 \times 1}{17 \times 3}$$

$$= \frac{9}{51} = \frac{9 \div 3}{51 \div 3} = \frac{3}{17}$$

How about another one?

$$1\frac{2}{7} \div 5$$

$$1\frac{2}{7} \div 5 = \frac{9}{7} \div \frac{5}{1} = \frac{9}{7} \times \frac{1}{5}$$

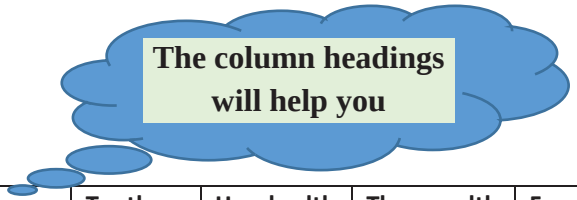
$$= \frac{9 \times 1}{7 \times 5} = \frac{9}{35}$$

Use the same trick you do when multiplying by changing everything into fractions and then go for it!

Sub-topic 3.4: Add, Subtract, Divide and Multiply Decimals

Activity 3.5: Fractions and decimals

In groups, copy and complete the table, by explaining how you have obtained the answer. The first three have been done for you



Tens	Ones	Tenth ($\frac{1}{10}$)	Hundredth ($\frac{1}{100}$)	Thousandth ($\frac{1}{1000}$)	Fraction	Percentage
		5			$\frac{1}{2}$	**50**
1	2	4			$12\frac{2}{5}$	1240
		2	5		$\frac{1}{4}$	25
		1	5	2		
		5				
						80
					$\frac{17}{20}$	
						64
		0	0	4		
					$\frac{3}{10}$	
4	0	3				

Sub-topic 3.5: Convert Fractions to Decimals and Vice Versa

A fraction like ¾ means *three quarters*

or three parts out of four

or three divided by four

3 divided by 4 equals 0.75

So, the fraction ¾ is equal to 0.75 in decimal.

Activity 3.6: In pairs, convert the following fractions into decimals

a) $^{2}/_{5}$
b) (b) $^{1}/_{20}$ (c) $^{5}/_{8}$ (d) $^{2}/_{9}$ (e) $^{1}/_{11}$
c) What do you notice about (d) and (e)?

Sub-topic 3.6: Identify and Classify Decimals as Terminating, Non-terminating and Recurring Decimals

Fractions like $^{3}/_{5}$, $^{1}/_{2}$, $^{3}/_{8}$ can be converted into decimals and they end or terminate: $^{3}/_{5}$ = 0.6, ½ = 0.5 and $^{3}/_{8}$ = 0.375.

Fractions like $^{2}/_{3}$, $^{2}/_{15}$, 1/11 do not end or terminate when converted into decimals, $^{2}/_{3}$ = 0.66666…, $^{2}/_{15}$ = 0.133333… and $^{1}/_{11}$ = 0.090909…
These decimals are referred to as **recurring decimals**

Exercise

1. Write the following fractions as decimals:

(a) $^{3}/_{8}$ (b) $^{7}/_{10}$ (c) $^{17}/_{50}$ (d) $^{13}/_{25}$

2. Write the following as fractions in their lowest terms:

(a) 0.25 (b) 0.08 (c) 0.35 (d) 0.125

3. Write the following fractions as recurring decimals:

(a)a $^{2}/_{11}$ (b) $^{1}/_{3}$ (c) $^{1}/_{6}$ (d) $^{7}/_{9}$

Sub-topic 3.7: Convert Recurring Decimals into Fractions

Recurring decimals can be converted into fractions.

Example: Convert this recurring decimal into a fraction: 0.333…

Note that the decimal repeats itself after one decimal place.

Let r = 0.333… (1)

Multiply both sides of the equation by 10 i.e. 10 x r = 10 x 0.333

10r = 3.333 (2)

Subtract equation (1) from equation (2):

That is, 10r = 3.333

- (r = 0.333)

9r = 3

r = $^{3}/_{9}$ = $^{1}/_{3}$.

Exercise

1. Convert the following recurring decimals into fractions
   a) 0.77---, b) 0.133--- , c)1.25656 ---, d) 0.2727 ---, e) 0.01313
2. Convert the following numbers into recurring decimals

a) $\frac{1}{3}$, b) $\frac{1}{9}$, c) $\frac{2}{6}$

Sub-topic 3.8: Convert Fractions and Decimals into Percentages and Vice Versa

Activity 3.7: Fraction percentage game

I am $\frac{7}{20}$	Who is 67%?	I am $\frac{67}{100}$	Who is 13%?	I am $\frac{13}{100}$	Who is 22%?
I am 11	Who is 5%?	I am 1	Who is72%?	I am 18	Who is 87%?
I am 87	Who is 4%?	I am 1	Who is 34%?	I am 8	Who is 42%?
I am 21	Who is 52%?	I am 13	Who is 45%?	I am 9	Who is 58%?
I am 29	Who is 64%?	I am 16	Who is32%?	I am 17	Who is2%?
I am 1	Who is 92%?	I am 23	Who is 98%?	I am 49	Who is 44%?
I am 11	Who is 82%?	I am 41	Who is 65%?	I am 13	Who is 14%?

From the fraction percentage game, identify the equivalent percentage for each fraction.

In your groups, use percentage to identify the smallest and largest fractions from the fraction percentage game.

Sub –topic 3.9 Calculate a Percentage of a Given Quantity

The percentage of a quantity can always be calculated in terms of percentage increase or percentage decrease.

Example 1: Find the 10% of 50,000

Solution: $^{10}/_{100}$ x 50,000 = 5,000.

Example 2: Opio had 60 goats. Now he has 63 goats. What is the percentage increase?

Solution: The increase in the number of goats is 63 – 60 = 3.
Percentage increase is $^{3}/_{60}$ x 100 = 5%.

Activity 3.8: The table below shows students' marks in two mathematics tests. For each one, calculate the percentage difference. Say if it is an increase or a decrease.

	Student	First Test	Second Test
(a)	Marion	50	45
(b)	James	40	52
(c)	Christina	20	35
(d)	Sarah	60	50

Sub-topic 3.10: Works out Real-life Problems Involving Percentages

Exercise

1. In a closing-down sale, a shop offers 50% cut of the original prices. What fraction is taken off the prices?
2. In a survey one in five people said they preferred a particular brand of Coca Cola. What is this figure as a percentage?
3. Peter pays tax at the rate of 25% of his income. What fraction of Peter's income is this?
4. When Carol was buying a house, she had to make a deposit of $\frac{1}{10}$ of the value of the house. What percentage was this?
5. I bought a coat in the January sales with $\frac{1}{3}$ price cut of the selling price. What percentage was taken off the price of the coat?
6. Adikinyi bought some fabric that was 1.75 metres long. How could this be written as a fraction?
7. A car park contains 20 spaces. There are 17 cars parked in the car park.
   a. What fraction of the car park is full?
   b. What fraction of the car park is empty?

Sub-topic 3.11: Identifying and classifying decimal as terminating, non-terminating and recurring decimals

Activity 3.6: Decimal as terminating, non-terminating and recurring decimals

In groups list some terminating, none terminating and recurring decimals. In pairs prove them. Compare your answers with the members of the group.

Situation of Integration

A primary school has two sections, that is, lower primary (P1-P4) and upper primary (P5-P7). The head teacher of the primary school needs to draw a timetable for both sections. The sections should start and end their morning lessons at the same time before break time, start and end their break time at the same time. The after break lessons should start at the same time. The lunchtime for both sections should start at the same time.

Support: The time to start lessons for the two sections is 8.00am. The duration of the lesson for the lower section is 30 minutes and that of the upper section is 40minutes.

Resources: Knowledge of fractions, percentages, natural numbers, factors, multiples, lowest common multiples, and the subjects taught in all classes and of time.

Task: Help the head teacher by drawing the timetable up to lunchtime for the two sections. How many lessons does each section have up to lunchtime?

Express the total number of lessons for the lower primary as a fraction of the total number of lessons for the whole School. (Consider lessons up to lunch time.)