Topic 1

NUMBER BASES

Key Words

base

binary

decimal

By the end of this topic, you should be able to:

i) identify numerals in base(s) up to base 16.
ii) identify place values of different bases using abacus.
iii) convert numbers from one base to another.
iv) manipulate numbers in different bases with respect to all four operations.

Introduction

I Am an ordinary person, how many fingers do I have on:

i) one hand?
ii) two hands?

If you have heaps of oranges of ten, twelve and fifteen, how many groups of tens, fives and fours do you get in each? And how many are remaining in each heap?

In order to answer the above questions, you can use **your knowledge of decimal place value to develop your understanding of numbers written in other bases.**

Sub-topic 1. 1: Identifying numbers of different bases on an abacus

In your primary education, you studied number bases such as bases five, two and ten (decimal base). Remember the numerals for all the various number bases you studied by doing the following activity:

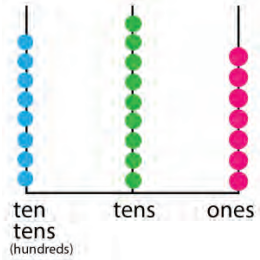
Activity 1. 1: Getting familiar with number bases

In your groups, identify situations in which you have ever used number bases in your life.

Real life situation	Base	Reason for the base chosen

Question: Which possible base does each abacus below represent?

a)



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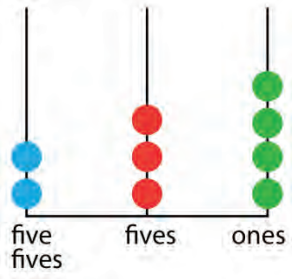

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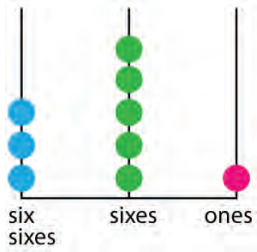

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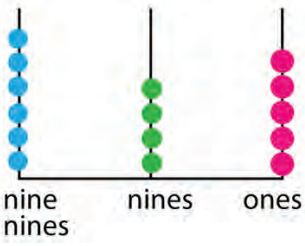


Activity 1.2: List the numerals for the following bases

In your groups, list the numerals for the following bases:

i) Two (Binary) ii) Three. iii) five iv) seven v) eight. vi) nine vii) eleven viii) twelve ix) sixteen

Now study the table below and fill in the gaps.

Base	Numerals
Two	0, 1
Three	0, 1, 2
Four	0, 1, -, 3
Five	0, - , 2, - , 4
Nine	0, 1, 2, - , 4, - , 6, - , 8
Twelve	0, - , 2, - , 4, - , - , 7, - , 9, - , e
Sixteen	0,1,2,3,4,5,-,-,-,9,t,e,-,-,-,-

Compare your answers and note what happens to the base number when writing the numerals used in a particular base. Give reasons.

Sub-topic 1. 2: Place Values Using the Abacus

You have already learnt how to represent numbers on an abacus. The representation of numbers on an abacus helps you to identify the place value of digits in any base.

Activity 1.3: Making abaci

In groups work in pairs to make different abaci, in different bases.

Compare your work with other members of the group .

Activity 1.4: Reading and stating the value of digits in bases

In groups, represent the following numbers on an abacus:

a. 123_{four}
b. 274_{ten}
c. 1312_{five}

Read and state what each digit in the numbers above represents on an abacus using the stated bases.

Exercise

State the place value of each numeral in the following numbers:

a) 321_{four} b) 354_{six} c) 247_{eight}

State the value of each numeral in the following numbers:

b) 567_{nine} b) 381_{twelve} c) 11010_{two}

Represent the following numbers on the abacus:

(a) 1101_{two} (b) 2102_{three} (c) 2021_{four} (d) 5645_{seven} (e) 8756_{nine}

1.3 Converting Numbers

Numbers can be converted from one base to another, and when you do this, you get the same numbers written in different bases.

You learnt how to convert from base ten to any other base.

Activity 1.5: Converting numbers from base ten to any other base
In groups, convert the following numbers in base ten to bases indicated: 456, 1321, 5693, 56 and 436.

(a) Five (b) Nine (c) Eight

You can also convert numbers from any base to base ten (decimal).

Example: Convert (a) 101_{two} (b) 324_{five} (c) 756_{eight} to base ten.

Solution:

(a) $101_{two} = (1x\ 2^2) + (0x2^1) + (1\ x\ 2^0) = 1x4 + 0x1 + 1x1 =$ $4+0+1 = 5$

(b) $324_{five} = (3x5^2) + (2x5^1) + (4x5^0) = 3x25 + 2x5 + 4x1 = 75+10+4 = 89$

(c) $756_{eight} = (7x8^2) + (5x8^1) + (6x8^0) = 7x64 + 5x8 + 6x1 = 448+40+6 = 494$

Activity 1.6: Converting numbers in a given base to another base
In pairs, discuss how to convert numbers in different bases to various bases in the exercise below.

Exercise

Convert the following numbers to the bases indicated: (a) 762_{eight} to base seven; (b) 234_{five} to base six; (c) 561_{seven} to base nine; (d) 654_{six} to base four; (e) 5432_{six} to twelve.

1.4: Operation on Numbers in Various Bases

James had two jackfruit trees in his compound. At one time one tree had 8 fruits ready and the other 7 fruits. He harvested them at the same time. He decided to put them in heaps of nine fruits. How many heaps of nine did he get and how many remained?

When you put the fruits in heaps of 9, you are adding in base 9.

Addition

The two jack fruit trees above had a total of 15 (that is 8 +7) ready fruits.

You can add numbers in various bases. For example, add the following numbers:

(a) 234_{five} to 23_{five} (b) 153_{seven} to 453_{seven}

Solution

(a)
$$\begin{array}{r} 235_{five} \\ +23_{five} \\ \hline 312_{five} \end{array}$$

(b)
$$\begin{array}{r} 153_{five} \\ +453_{five} \\ \hline 636_{five} \end{array}$$

Exercise: Add the following numbers:

(a)321_{four} to $122_{four.}$ (b) 456_{seven} to 342_{seven}

(c) 764_{eight} to $361_{eight.}$ (d) 210_{three} to 211_{three}

Subtraction

Subtraction in other bases is done in the same way it is done in base ten.

Examples: Subtract (a) 342_{eight} from 567_{eight} (b) 432_{six} from 514_{six}

Solution:

(a)
$$\begin{array}{r} 567_{eight} \\ -342_{eight} \\ \hline 224_{eight} \end{array}$$

(b)
$$\begin{array}{r} 514_{six} \\ -\ 432_{six} \\ \hline 224_{six} \end{array}$$

Exercise

Subtract the following numbers in the given bases:

(a) 351_{six} from 510_{six}(b) 672_{nine} from 854_{nine}
(c) 845_{twelve} from $t23_{twelve}$(d) 231_{five} from 421_{five}

Multiplication

Multiplication is done in the way it is done in base ten.

Example: Multiply 423_{five} by 12_{five}

Solution

$$\begin{array}{r} 423 \\ \times 12 \\ \hline 1401 \\ +4230 \\ \hline 11131 \end{array}$$

Exercise:

Multiply the following:

(a) 241_{five} by $13_{five.}$ (b) 345_{six} by 24_{six}

(c) 534_{seven} by 123_{seven}. (c) 156_{eleven} by 534_{eleven}

Division

The most common method of dividing numbers in different bases is by converting the numbers to base ten first and after division, you can convert the answer to the given base.

Example: Divide 1111_{two} by 101_{two}

Solution: Convert 1111_{two} and 101_{two} to base ten

$1111_{two} = (1x2^3) + (1x2^2) + (1x2^1) + (1x2^0)$

$= 8 + 4 + 2 + 1$

$= 15.$

$101_{two} = (1x2^2) + (0x2^1) + (1x2^0)$

$= 4 + 1 = 5_{ten}$

Therefore, 1111_{two} divided 101_{two} is the same as 15 divided 5.

$15 \div 5 = 3$

$3_{ten} = 3 \div 2 = 1$ remainder $1 = 11_{two}$

Therefore, $1111_{two} \div 101_{two} = 11_{two}$

Exercise:

1. Add: (a) 654_{seven} to 514_{seven} (b) 278_{nine} to 756_{nine}
2. Subtract: (a) 412_{six} from 554_{six} (b) 435_{eight} from 764_{eight}
3. Multiply: (a) 1121_{three} by 212_{three} (b) 312_{four} by 122_{four}
4. Divide: (a) 100011_{two} by 111_{two} (b) 150_{nine} by 20_{nine}

Activity 1.6: Operations on numbers with mixed bases

In your groups work in pairs discuss how you would carry out the four mathematical operations on numbers with mixed bases by getting your own examples. Compare your answers with other members of the group.

Number Game: You are given four boxes containing numbers in base ten. The boxes are labelled Box 1, Box 2, Box 3 and Box 4.

9 1 15 7

Box 1

6 14 2 7 15

Box 2

15 14 6 12 4 7

Box 3

15 14 9 12

Box 4

Task: Working in groups, select one number from any of the boxes given. Your mathematics teacher will ask you whether the number you selected appears in Box 1, Box 2, Box 3 and Box 4. From the responses you give, the teacher will tell you the number you selected. Discuss how the teacher was able to tell you the number you had selected.

Situation of Integration

A community is hit by famine and the government decides to give each member in the household a potato to solve their problem of hunger.

Support: Each package contains an equal number of potatoes of five.

There are 10 households in the community with 3, 5, 7, 4, 6, 5,8,12, 13 members respectively.

Resources: Knowledge of Bases, knowledge of mathematical operations

Task: Determine the number of packages of potatoes the government will take to that community. In case there are remaining potatoes, discuss what the government should do with them.

Topic 2:
WORKING WITH INTEGERS

Key Words

positive, negative, BODMAS, LCM, HCF

By the end of this topic, you should be able to:

i) identify, read and write natural numbers as numerals and words in million, billion and trillion.
ii) differentiate between natural numbers and whole numbers/integers.
iii) identify directed numbers.
iv) use directed numbers (limited to integers) in real life situations.
v) use the hierarchy of operations to carry out the four mathematical operations on integers.
vi) identify even, odd, prime and composite numbers.
vii) find the prime factorisation of any number.
viii) relate common factors with HCF and multiples with LCM.
ix) work out and use divisibility tests of some numbers.

Introduction

Sarah was sent to a shop up the hill to buy 1kg of sugar, a packet of salt and a packet of tealeaves. She was given UGX. 5,000 note but all items cost her UGX. 6,500. How much money did Sarah owe the shopkeeper?

In your day-to-day life, you use numbers to count items, to keep information, to transact business and many others. Since you use numbers in your day-to-today situations, knowledge of integers will be helpful to you.

Subtopic 2.1: Natural Numbers

In lower primary, you learnt counting items using numbers one, two, three ---. In mathematics these numbers are called counting or natural numbers.

When zero is included in the set of natural numbers, they become whole numbers.

For example: N = $\{1,2,3,4,5,------\}$ This is a set of natural numbers.

W = $\{0,1,2,3,4,5.------\}$ This is a set of whole numbers.

Activity 2.1: Natural numbers

There is a box and a board. In the box, there are number cards: some have numbers in figures and others in words. While the board has two sections: one section for natural and the other for non-natural numbers.

In groups, pick a card and place it in the appropriate section of the board.

Is it possible for a number to belong to two sections?

What can you say about the two categories of the numbers picked?

Where in real-life situations do we find such numbers?

Activity: 2.2: Writing and reading numbers

There are two boxes. In one box, number cards are written in figures and the others in words.

In groups, a member picks one card from one of the boxes. After all the cards have been picked, one member displays his/her card; then the others check their cards, and the matching card is displayed.

Exercise

Write the following in words:

1. 3,800
2. 8,008,008
3. 606,520,060
4. 9,000,909,800
5. 4,629,842,003
6. 1,629,284,729,000

Write the following in figures:

7. Six hundred two million eight thousand and eight
8. Two billion eighty-nine million four thousand seven
9. One trillion two hundred fifty billion eight hundred seventy-five million three hundred sixty thousand
10. State the value of digit four in the following numbers.
    i) 7,462,300,800
    ii) 24,629,293,005

Sub topic 2.2: Differentiating between natural numbers and whole numbers/integers

Activity 2.3: Relating natural numbers and integers

In groups, **read** the text below and answer the questions that follow:

Two learners—Mary and Joy—went to the school canteen to buy some snacks for their breakfast. Joy bought 3 pancakes at UGX.200 each and 1 ban at UGX. 300.

Mary checked her bag and found out that her money was stolen. She borrowed some money from Joy. She bought four 4 pancakes and 2 bans.

Questions

i) Which of the two learners had more money?
ii) How much money did Mary borrow from Joy?
iii) Peter said that Mary had negative UGX. 1400. Was he correct? Give reasons for your answer.

Sub-topic 2.3: Use Directed Numbers (Limited to Integers) in Real-life Situations

Activity 2.4: Integers in real-life situations

In groups, read the story below and answer the questions.

Once upon a time, there lived an old woman. She had hot and cold stones and a big pot of water. If she put one hot stone in the water, the temperature of the water would rise by 1 degree. If she took the hot stone out of the water again, the temperature would go down by 1 degree.

Question 1

If the temperature of the water is 24 degrees and the old woman adds 5 hot stones, what is the new temperature of the water?

Now imagine that the temperature of the water is at 29 degrees. The old woman takes a spoon and takes out 3 of the hot stones from the pot.

Question 2

What is the temperature of the water when the old woman removes 3 hot stones? Explain your answer.

The old woman also had cold stones. If she adds 1 cold stone to the water, the temperature goes down by 1 degree. The temperature of the water was 26 degrees. Then the old woman added 4 cold stones.

Question 3

What is the temperature of the water after the old woman added 4 cold stones? Give a reason for your answer.

Just like the old woman could remove the hot stones and the temperature would decrease she could also remove the cold stones.

Question 4

Imagine that the temperature of the water was 22 degrees and the old woman removes 3 cold stones. What happens to the temperature of the water?

What is the new temperature of the water? Explain your answer.

Activity 2.5: Real-life situations

In groups, get a cup of hot water and a thermometer. Identify a timekeeper in your group. One of you reads the temperature on the

thermometer and the other members record in their notebooks. Put the thermometer back into the hot water and after 5 minutes take the reading on the thermometer. Repeat this at same interval of 5 minutes for duration of 25 minutes.

Question 1

What is the change in temperature between the first reading and the second reading?

Question 2

What is the change in temperature between the 2nd and 3rd reading?

Question 3

What is the change in temperature between the 3rd and the 4th reading?

Question 4

What is the difference in temperature between the 4th and the 5th reading?

Sub-topic 2.4: Use the Hierarchy of Operations to Carry out the Four Mathematical Operations on Integers

Activity 2.6: Operations on integers

In groups, read the text below and answer the questions after.

Sarah moved 5 steps to the right from a fixed point. Then she moved 9 steps to the left.

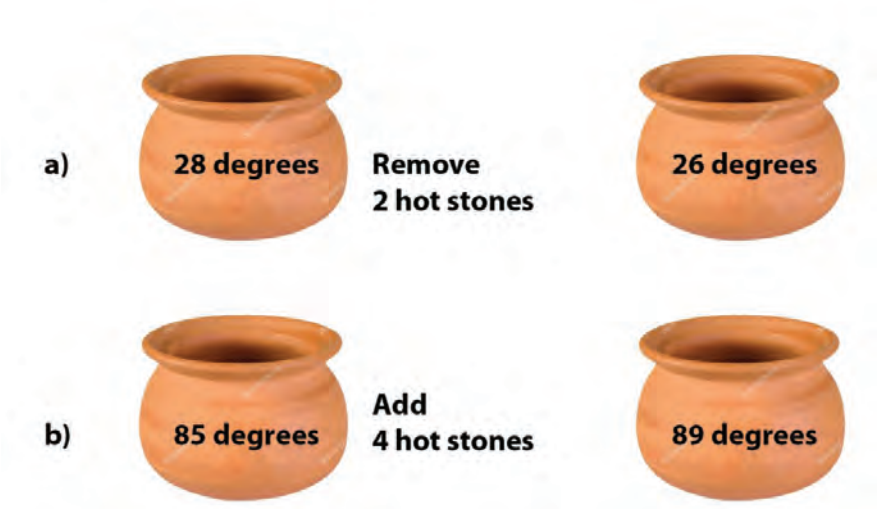
Question 1

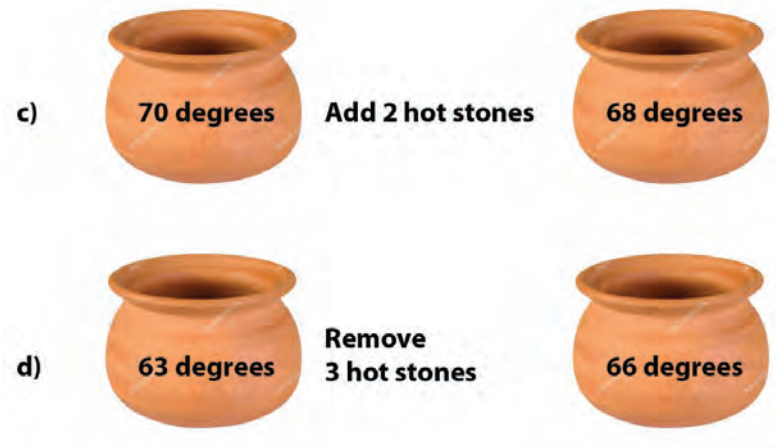
How far is Sarah from the fixed point?

Question 2

Peter gave his answer as 4 steps to the left of the fixed point and John as -4 (negative 4). Who is correct? Give reasons for your answer.

Example 1



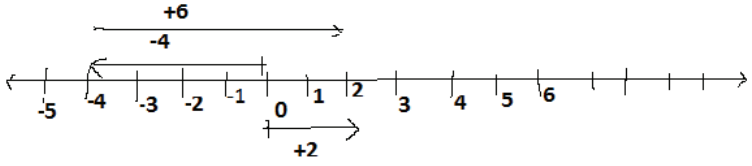


Example 2: A group of learners of Geography went for a tour to Kabale. They found out that the temperature at one time was 13^0C. At around mid-night the temperature was 10^0C. By how many degrees had the temperature dropped?

Solution: 10^0C - 13^0C = - 3^0C

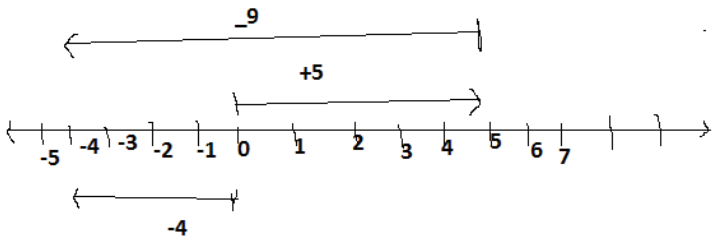
Example 2: Using a number line work out:

a) – 4 + +6



-4 + +6 = + 2

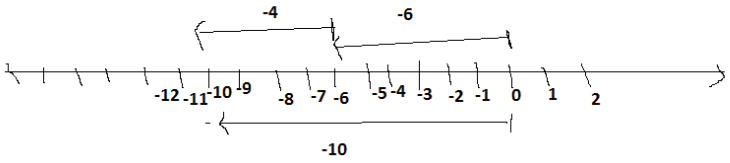
b) **+5 + - 9**

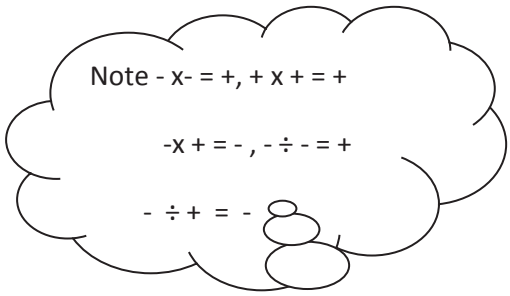


+5 + -9 = -4

c) **-6 - 4 = -6 + - 4**

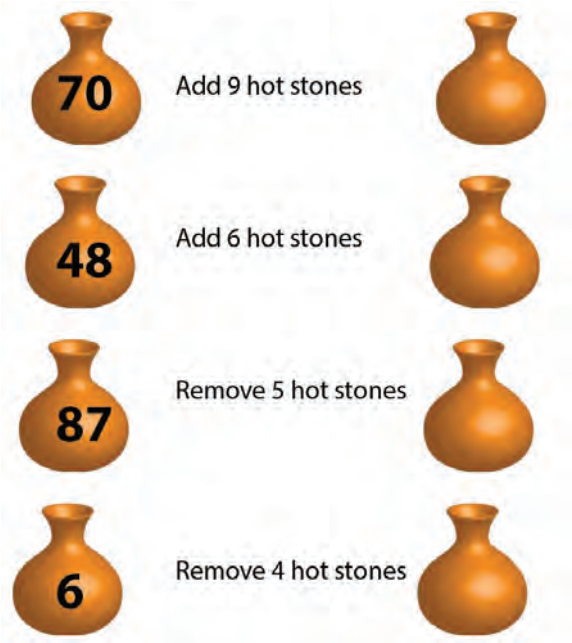
-6 - 4 = -6 + -4 = -10

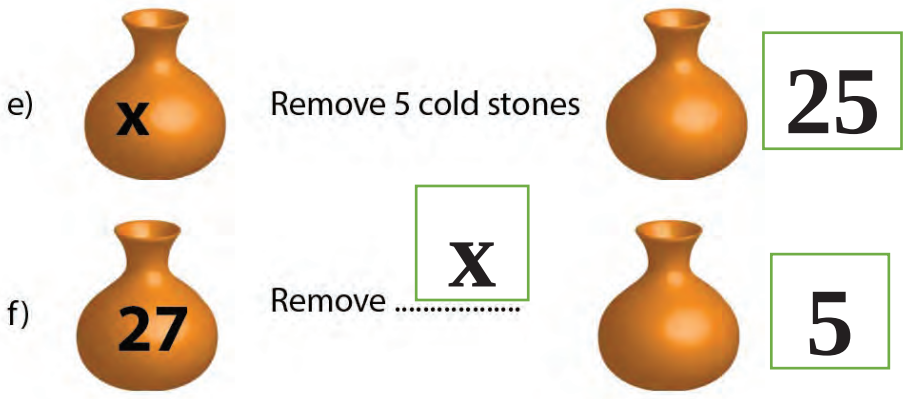




Exercise

1. Work out the following in degrees:





2. Work out the following:

a) 8 + -6

b) 61 + + 7

c) 49 - - 30

d) 77 - + 50

e) -15 + + 20

f) -3 - - 13

3. 2.Using a number line work out:

a) -2 ++ 3

b) +5 + - 6

c) – 8 - -5

4. A national park guide on one of the mountains in East Africa recorded the temperature as 15^0C one day. At midnight the temperature was -7^0C. By how many degrees had the temperature fallen?
5. Write down the next 3 terms in the sequence - 9, -7, - 5, -3, - , - ; -
6. Look at the sequence of the numbers:

-1, 3,-9, +27, -, - , -

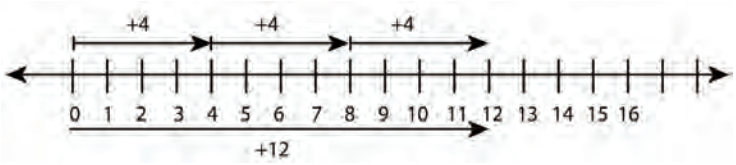
Alex said the next three terms are +9, -36 and -729. Is Alex correct? Give reasons for your answer.

Multiplication and Division

Multiplication such as +4 × + 3 or -4 × + 3 are interpreted as repeated addition of positive or negative numbers.

+4 × + 3 = + 4 + +4 + +4 = +12

-4 × +3 = -4 + -4 + -4 = -12



Negative

3 × 3 = 9

3 ×- 3 = -9

-3 × -3 =9

Justification of the above is as follows:

$3 \times 3 = 9$

$3 \times 2 = 6$

$3 \times 1 = 3$

$3 \times 0 = 0$

$3 \times -1 = -3$

$3 \times -2 = -6$

$3 \times -3 = -9$

Now reduce the first multiplier

$3 \times -3 = -9$

$2 \times -3 = -6$

$1 \times -3 = -3$

$0 \times -3 = 0$

$-1 \times -3 = 3$

$-2 \times -3 = 6$

$-3 \times -3 = 9$

The justification shows that any number multiplied by zero is zero; that a positive number multiplied by a positive number is a positive; a negative number multiplied by a positive number is a negative, and a negative number multiplied by a negative is a positive.

Multiplication and division have the same rules:
A negative number divided by a positive and a positive number divided by a negative number is a negative, Also a negative number divided by a negative is a positive.

Example

+ 4 × -3 = -12

-12 ÷ - 3 = +4

- 12 ÷ +4 = - 3

Note: Rules of integers

a) Positive number multiplied by a positive number is a positive.
b) Negative number multiplied by a positive number is a negative.
c) Negative number multiplied by a negative is a positive.
d) Negative number divided by a positive is a negative
e) Positive number divided by a negative is a negative.
f) Negative number divided by a negative is a positive.

Exercise

Work out

1. - 2 ×+ 4 × -3

2. -4 ×+2 × - 3

3. -3× -5 × +2

4. -12 ×-5 ÷ +6

5 -15 ÷ 5 × -4

6. -24 × + 4 ÷+2

7. In a certain test a correct answer scores 3marks and an incorrect answer, the child gets a penalty of two marks deducted. Joy guessed all the answers. She got 6 correct and 4 wrong. Work out her total marks.

8. Simplify +6 - +7 ÷ +4 + + 6 × +7

9. Work out 7 of 13 – (18 ÷ 6 +3) ÷ (9 × 3 -25)

10. 56 - (38 - 35 ÷5 +2)

11. 69 ÷ (6 + (3 × 8 -7))

12. 4 of (5 + 2) - 2 (3 + 7) ÷ 5

Sub-topic 2.5: Identify Even, Odd, Prime and Composite Numbers

Natural numbers can be classified into various groups of numbers. In your primary education, you learnt numbers such as even, odd, prime and composite.

Activity 2.6: Identifying even, odd, prime and composite numbers

Each group is given a box containing number cards. In your groups pick the card and read the number. Identify which group of numbers it belongs to by filling the table below.

No	Odd	Prime	Even	Composite

Question 1

Are there numbers that belong to more than one group?

Question 2

How do you identify that a number is:

a) odd
b) even
c) prime
d) composite

Sub-topic 2.6: Find the Prime Factors of any Number

In your primary education you studied multiples and factors of numbers. When two numbers are multiplied together, the product is called multiple. The two numbers multiplied together are called factors of the multiple.

Note: A multiple has two or more factors.

For example: The factors of 12 are $(1 \times 12), (2 \times 6)$, and (3×4); hence, the factors of 12 are $\{1,2,3,4,6,12\} = F_{12} = \{1,2,3,4,6,12\}$

The multiples of 3 are $\{3,6,9,12,15,18,21---\}$ = M_3 = $\{3,6,9,12,15,18,21--\}$

Exercise

Find the factors of the following:

1. 42
2. 56
3. 36
4. 108

Find the multiples of the following:

5. 7
6. 12
7. 9
8. 5

Note: A factor of a number which is a prime number is called its prime factor. For example the factors of 36 are $\{1,2,3,4,6,9,12,36\}$

Qn. What are the prime factors of 36?

Qn. Write 36 as a product of its prime factors.

Answer:

Prime Factor	Number
2	36
2	18
3	9
3	3
	1

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

This approach of determining prime factors is called prime factorisation.

This can be written in power notation.

Exercise

Find the prime factors of the following numbers. Give your answer in power form.

1. 108
2. 288
3. 180
4. 1232
5. 993
6. 2145

Sub-topic 2.7: Relate Common Factors with HCF and Multiples with LCM

A number can have one or more common factors; for example, 2 and 4 are common factors of 8 and12. However, the highest common factor is 4. Therefore, the highest common factor (HCF) of 8 and 12 is 4.

Activity 2.7: Highest common factor

In groups, find the HCF of the following:

i) 54, 48
ii) 42 ,63 ,105
iii) 132, 156,204,228

Sub-topic 2.8: Work Out and Use Divisibility Tests of Some Numbers

Activity 2.8: Identifying divisibility tests for some numbers

1. In your groups, pick a number card and determine which numbers on the chart divides it. Write a number under its divisor.
2. What can you say about the numbers under each divisor? Give reasons for your answers.
3. The relationship between the dividend and the divisor leads to divisibility t**ests.**

Exercise

Given the following numbers:

12, 132, 1212, 3243, 1112, 81, 18, 27, 279, 2580, 5750

Find out which of them are divisible by:

a) 3 b) 4 c) 6 d) 9 e) 10

Exercise

Find the HCF the following:

1. $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 11$
2. $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$
3. 2. $2^3 \times 3^2 \times 5^2$, $2^5 \times 3^5 \times 5^2$
4. 36, 60, 84

4. A rectangular field measures 616m by 456m. Fencing posts are placed along its sides at equal distances. What will be the distance between the posts if they are placed as far apart as possible? How many posts are required?

Sub-topic 2.9: Least Common Multiple (LCM)

In the previous section of multiples and factors you learnt about multiples of numbers. For example, the multiples of 5 are 5, 10, 15,20,25,30, 35, 40, 45, 50, 55, 60, 65, 70, 75 ------. The multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77 ------. From the above example, 35 and 70 are common multiples of 5 and at the same time of 7. However, 35 is smaller than 70, therefore, 35 is the least common multiple of 5 and 7.

There is another approach of getting LCM of numbers without listing the multiples of the numbers.

Example

Find the LCM of 8 and 12

2	8	12
2	4	6
2	2	3
3	1	3
	1	1

$2 \times 2 \times 2 \times 3 = 24$

The LCM of 8 and 12 is 24.

Activity 2.9: In your groups, find the LCM of the following:

a) 28, 42 ,98
b) 35,48 ,56, 70

Exercise

Find the LCM of the following numbers:

1. 14, 21
2. 18, 24, 96
3. 49, 84, 63
4. 60, 72, 84, 112
5. Determine the smallest sum of money out of which a number of men, women and children may receive UGX. 75, Ush.90 and Ush.120 each.