MATHS:

Change of Subject of formulae

formulae involving squares and square roots

ex.1. Make r the subject of the formula

$$F = \frac{mr^2}{K}$$

multiply both sides by K or cross multiply

$$K \times F = \frac{Mr^2}{K} \times K$$

$$KF = mr^2$$

divide both sides by m

$$\frac{KF}{M} = \frac{Mr^2}{M}$$

Take square root of both sides.

$$\sqrt{\frac{KF}{M}} = \sqrt{r^2}$$

$$\sqrt{\frac{KF}{M}} = r \quad \therefore r = \sqrt{\frac{KF}{M}} \quad \text{or} \quad r = \pm\sqrt{\frac{KF}{M}}$$

ex. 2. Express U in terms of V, a and S given $V^2 = U^2 + 2aS$

$$V^2 = U^2 + 2aS$$

Subtract 2aS from both sides

$$V^2 - 2aS = U^2 + 2aS - 2aS$$

$$V^2 - 2aS = U^2$$

Find / take square root of both sides

$$\sqrt{V^2 - 2aS} = \sqrt{U^2}$$

$$\sqrt{V^2 - 2aS} = U \quad \therefore U = \sqrt{V^2 - 2aS}$$

$$\therefore U = \pm\sqrt{V^2 - 2aS}$$

ex.3 Make r the subject of $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\sqrt{\frac{3V}{\pi h}} = \sqrt{r^2}$$

$$\sqrt{\frac{3V}{\pi h}} = r \quad \therefore \quad r = \sqrt{\frac{3V}{\pi h}}$$

NOTE The answer can be $r = \sqrt{\frac{3V}{\pi h}}$

or $r = \pm\sqrt{\frac{3V}{\pi h}}$ as every number or expression has 2 square roots a positive and a negative

ex. 4 : Express L in terms of T, π and g

if $T = 2\pi\sqrt{\frac{L}{g}}$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Divide both sides by 2π

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

Square both sides

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{g}}\right)^2 \qquad \frac{T^2}{4\pi^2} = \frac{L}{g}$$

multiply g on both sides

$$g \times \frac{T^2}{4\pi^2} = \frac{L}{g} \times g$$

$$\frac{T^2 g}{4\pi^2} = L \quad \text{or} \quad L = \frac{T^2 g}{4\pi^2}$$

~~ex. 5.~~ Alternatively

$$T = 2\pi\sqrt{\frac{L}{g}}$$

square both sides

$$T^2 = \left(2\pi\sqrt{\frac{L}{g}}\right)^2$$

$$T^2 = 4\pi^2 \frac{L}{g}$$

Multiply both sides by g or cross multiply

$$T^2 g = 4\pi^2 L$$

Divide both sides by $4\pi^2$

$$\frac{T^2 g}{4\pi^2} = \frac{4\pi^2 L}{4\pi^2}$$

$$\frac{T^2 g}{4\pi^2} = L \quad \therefore \quad L = \frac{T^2 g}{4\pi^2}$$

ex. 5. Express x in terms of a and b when $b = 3\sqrt{a-x^2}$

$b = 3\sqrt{a-x^2}$

Square both sides

$b^2 = \left(3\sqrt{a-x^2}\right)^2$

$b^2 = 9(a-x^2)$

$\frac{b^2}{9} = \frac{9(a-x^2)}{9}$

$\frac{b^2}{9} = a-x^2$

$x^2 = a - \frac{b^2}{9}$

$\sqrt{x^2} = \sqrt{\frac{9a-b^2}{9}}$ $\quad x = \sqrt{\frac{9a-b^2}{9}}$

or $\quad b = 3\sqrt{a-x^2}$

$\frac{b}{3} = \frac{3}{3}\sqrt{a-x^2}$

$\left(\frac{b}{3}\right)^2 = \left(\sqrt{a-x^2}\right)^2$

$\frac{b^2}{9} = a-x^2$

$x^2 = a - \frac{b^2}{9} \longrightarrow \sqrt{x^2} = \sqrt{\frac{9a-b^2}{9}}$

$\sqrt{x^2} = \sqrt{a - \frac{b^2}{9}}$

$x = \sqrt{a-\frac{b^2}{9}}$ $\quad x = \sqrt{\frac{9a-b^2}{9}}$

ex.6 make b the subject of the equation

$t = 20 + \sqrt{a-b^2}$

take 20 to the LHS

$t - 20 = \sqrt{a-b^2}$

Square both sides

$(t-20)^2 = (\sqrt{a-b^2})^2$

$(t-20)^2 = a - b^2$

$b^2 = a - (t-20)^2$

Take square root of both sides

$\sqrt{b^2} = \sqrt{a-(t-20)^2}$

$b = \sqrt{a-(t-20)^2}$

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ex.7 Make T the subject of

$K = \sqrt{\frac{PT}{S+T}}$

Square both sides

$K^2 = \left(\sqrt{\frac{PT}{S+T}}\right)^2$

$K^2 = \frac{PT}{S+T}$

$K^2(S+T) = PT$

$K^2S + K^2T = PT$

$K^2T - PT = -K^2S$

$\frac{T(K^2-P)}{K^2-P} = \frac{-K^2S}{K^2-P}$

$T = \frac{-K^2S}{K^2-P}$

or

$K^2S = PT - K^2T$

$\frac{K^2S}{P-K^2} = \frac{T(P-K^2)}{P-K^2}$

$\frac{K^2S}{P-K^2} = T$

$T = \frac{K^2S}{P-K^2}$

ex. 8 make h the subject of

$d = \frac{\sqrt{3h}}{2}$ or $d = \frac{\sqrt{3h}}{2}$

$2d = \sqrt{3h}$

Square both sides

$(2d)^2 = (\sqrt{3h})^2$

$d^2 = \left(\frac{\sqrt{3h}}{2}\right)^2$

$\frac{4d^2}{3} = \frac{3h}{3}$

$d^2 = \frac{3h}{4}$

$\frac{4d^2}{3} = h$

$\frac{4d^2}{3} = \frac{3h}{3}$

$\therefore h = \frac{4}{3}d^2$

$\frac{4d^2}{3} = h$

$\therefore h = \frac{4d^2}{3}$

ex. 9. make r the subject of the formula $V = \frac{4}{3}\pi r^3$

$3V = \frac{4}{3}\pi r^3 \times 3$

$\frac{3V}{4\pi} = \frac{4\pi r^3}{4\pi}$

Take cube root of both sides.

$\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3}$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

ex. 10 Express w in terms of A and K

when $A = \sqrt[4]{\dfrac{K+2W}{1-2W}}$

raise both sides to power 4

$$A^4 = \left(\sqrt[4]{\frac{K+2W}{1-2W}}\right)^4$$

Note:

$(\sqrt[4]{K})^4$

$= \sqrt[4]{K} \times \sqrt[4]{K} \times \sqrt[4]{K} \times \sqrt[4]{K}$

$= \sqrt[4]{K^4}$

$= K$

$(\sqrt[4]{3})^4 = \sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3}$

$= \sqrt[4]{81}$

$= 3$

$$A^4 = \frac{K+2W}{1-2W}$$

$$A^4(1-2W) = K + 2W$$

$$A^4 - 2WA^4 = K + 2W$$

$$A^4 - K = 2W + 2WA^4$$

$$A^4 - K = \cancel{2A^4}\, W(2 + 2A^4)$$

$$\frac{A^4-K}{2+2A^4} = \frac{W(2+2A^4)}{2+2A^4}$$

$$\frac{A^4-K}{2+2A^4} = W \quad \therefore W = \frac{A^4-K}{2+2A^4}$$

$$\text{or } W = \frac{A^4-K}{2(1+A^4)}$$

ACTIVITY

1(a) Make r the subject of $F = \dfrac{GMH}{r^2}$

(b) make x the subject of $y = 4ax^2$

(c) Express t in terms a and u in

$S = ut + \frac{1}{2}at^2$

(d) Re-arrange $x^2 + y^3 = r^2$ make y the subject

2. Re-arrange the following formulae making the stated letter the subject

(a) $T = 2\pi\sqrt{\frac{J}{MH}}$ (M)

(b) $r = w\sqrt{(a^2 - x^2)}$ (x)

(c) $d = \sqrt{\frac{c-1}{c+1}}$ (c)

(d) $T = \frac{k}{2a}\sqrt{\frac{r}{a-b}}$ (b)

e $\frac{\sqrt[3]{x}}{2} = y$ (x)

Making the subject of (change of the subject) and substitution (combination)

ex1. Express a in terms of v, u and t. Hence determine the value of a when v = 20, u = 4 and t = 2 given that v = u + at

$v = u + at$

$\frac{v-u}{t} = \frac{at}{t}$

$a = \frac{v-u}{t}$

hence $a = \frac{v-u}{t}$

a = ?
v = 20
u = 4
t = 2

$a = \frac{20-4}{2}$

$a = \frac{16}{2}$ $\therefore a = 8$

Note

Hence means continue from where you have stopped. (use the solution got and continue)

- when hence is not put into consideration

ex.

$V = U + at$ $\quad V = 20 \quad U = 4 \quad t = 2$

$20 = 4 + 2a$

$20 - 4 = 2a$

$\frac{16}{2} = \frac{2a}{2}$

$a = 8$

refer to ex1 if the value of a is got minus considering HENCE then the answer is ~~corr~~ wrong.

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ex. 2. Given that $a = \frac{b+x}{b-x}$ make x the subject of the formula, hence find x if $b = -2$ and $a = 1$

$a = \frac{b+x}{b-x}$

$a(b-x) = b+x$

$ab - ax = b + x$

$ab - b = x + ax$

$ab - b = x(1+a)$

$\frac{ab-b}{1+a} = \frac{x(1+a)}{1+a}$

$\frac{ab-b}{1+a} = x$ or $x = \frac{ab-b}{1+a}$

or $x = \frac{b(a-1)}{1+a}$

Hence $x = ?$ $\quad b = -2 \quad a = 1$

$x = \frac{b(a-1)}{1+a}$

$x = \frac{-2(1-1)}{1+1}$

$x = \frac{-2 \times 0}{2}$

$x = \frac{0}{2}$

$x = 0$

ex. 3 Given that $Q = I^2RT$ make I the subject, Hence work out I when $Q = 1000$ $t = 20$ $R = 2$

$$\frac{Q}{RT} = \frac{I^2RT}{RT}$$

$$\sqrt{\frac{Q}{RT}} = \sqrt{I^2}$$

$$I = \sqrt{\frac{Q}{RT}} \quad \text{or} \quad I = \pm\sqrt{\frac{Q}{RT}}$$

Hence

$I = ?$
$Q = 1000$
$T = 20$
$R = 2$

$$I = \pm\sqrt{\frac{Q}{RT}}$$

$$I = \pm\sqrt{\frac{1000}{2 \times 20}}$$

$$= \pm\sqrt{\frac{1000}{40}}$$

$$= \pm\sqrt{25}$$

$$= \pm 5$$

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ex. 4. Make x the subject of $y = 4ax^2$ Hence calculate x when $y = 72$ and $a = 2$

$$y = 4ax^2$$

$$\frac{y}{4a} = \frac{4ax^2}{4a}$$

$$\sqrt{\frac{y}{4a}} = \sqrt{x^2}$$

$$\sqrt{\frac{y}{4a}} = x$$

Hence

$x = ?$
$y = 72$
$a = 2$

$$x = \sqrt{\frac{y}{4a}}$$

$$x = \sqrt{\frac{72}{4 \times 2}}$$

$$x = \sqrt{\frac{72}{8}}$$

$x = 3$ or $x = -3$

or $x = \pm 3$

ex. 5. Express L interms of T, g and π
Hence
Look for L given that T = 100 g = 1 and leave π as π

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{L}{g}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{g}}\right)^2$$

$$\frac{T^2}{4\pi^2} = \frac{L}{g}$$

$$g \times \frac{T^2}{4\pi^2} = \frac{L}{g} \times g$$

$$\frac{T^2 g}{4\pi^2} = L$$

Hence $L = ?$ $g = 1$ $T = 100$ $\pi = \pi$

$$L = \frac{T^2 g}{4\pi^2}$$

$$L = \frac{100^2 \times 1}{4\pi^2}$$

$$L = \frac{10000}{4\pi^2}$$

$$L = \frac{2500}{\pi^2}$$

ex. 6. Make T the subject of

$L = \sqrt[3]{\frac{m^2 - T^2}{px}}$ Hence evaluate T

when $m = 10$ $p = 2$ $x = \frac{2}{3}$

and ~~t = 3~~ $L = 3$

$$(L)^3 = \left(\sqrt[3]{\frac{m^2 - T^2}{px}}\right)^3$$

$$L^3 = \frac{m^2 - T^2}{px}$$

$$L^3 px = m^2 - T^2$$

$$\sqrt{T^2} = \sqrt{m^2 - L^3 px}$$

$$T = \pm \sqrt{m^2 - L^3 px}$$

Hence $T = ?$ $m = 10$, $P = 2$ $x = \frac{2}{3}$ $L = 3$ ~~T = 3~~

So $T = \pm \sqrt{m^2 - L^3 px}$

$$T = \pm \sqrt{10^2 - (3^3 \times 2 \times \frac{2}{3})}$$

$$= \pm \sqrt{100 - (\cancel{3} \times 3 \times 3 \times 2 \times \frac{2}{\cancel{3}})}$$

$$= \pm \sqrt{100 - 36}$$

$$= \pm \sqrt{64}$$

$$= \pm 8$$

ACTIVITY

1. Make C the subject of $\frac{9}{5}C = F - 32$

Hence find C when $F = 212$

2. Express h in terms of A, π and r when $A = 2\pi r(r+h)$. Hence determine h if $\pi = \frac{22}{7}$, $r = 14\,cm$ and $A = 4488\,cm^2$

3. Given that $L = 4\sqrt{px^2 + a}$ make x the subject and Hence Look for the values of x

4. Find u in terms of v, a and s if $v^2 = u^2 + 2as$. Hence look for u if $v = 30$ $a = 8$ $s = 44$

5. ~~Make r the subject of the formula $V = \frac{4}{3}\pi r^3$ Hence work out the value~~ of

5. Make n a subject of the formula $Q = v + \sqrt{\frac{n}{R}}$ hence find the value of n when $R = \frac{1}{2}$ $b = 3$ and $v = 7$

6. Make p the subject of the equation $\sqrt[3]{\frac{p^2}{m}} = y$ hence work out p given that $m = 2$ and $y = 3$.