

SENIOR TWO LESSON NOTES

1

TOPIC EQUATIONS AND FORMULAE

An equation like $2x - 1 = 3$ is called a linear equation. To solve an equation means to find the real number value of the unknown which makes the equation true or equal on both sides.

ex. 1. Solve for x

$$2x - 1 = 3$$

Add 1 to both sides of the equation

$$2x - 1 + 1 = 3 + 1$$

$$2x + 0 = 4$$

$$2x = 4$$

Divide both sides of the equation by 2

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

ex. 2. Solve $4 - 4x = 9 - 12x$

add $12x$ to both sides of the equation

$$4 - 4x + 12x = 9 - 12x + 12x$$

$$4 + 8x = 9$$

subtract 4 from both sides of the equation

$$4 - 4 + 8x = 9 - 4$$

$$8x = 5$$

Divide both sides by 8

$$\frac{8x}{8} = \frac{5}{8}$$

$$x = \frac{5}{8}$$

Equations with brackets

ex. 1. Solve $3 - (3m - 7) = 43$

Remove brackets

$$3 - 3m + 7 = 43$$

$$-3m + 10 = 43$$

Take 10 to the RHS (it has been positive)

So it becomes negative

$$-3m = 43 - 10$$

$$-3m = 33$$

Divide both sides by -3

$$\frac{-3m}{-3} = \frac{33}{-3}$$

$$m = -11$$

CHECKING.

$$3 - (3m - 7) = 43$$

$$3 - (3(-11) - 7) = 43$$

$$3 - (-33 - 7) = 43$$

$$3 + 33 + 7 = 43$$

$$43 = 43$$

$$\text{LHS} = \text{RHS}$$

ex. 2. Solve $3(4c - 7) - 4(4c - 1) = 0$

Remove brackets

$$12c - 21 - 16c + 4 = 0$$

Collect like terms

$$12c - 16c - 21 + 4 = 0$$

$$-4c - 17 = 0$$

Take -17 to the other side and it becomes positive

$$-4c = 17$$

Divide both sides by -4

$$\frac{-4c}{-4} = \frac{17}{-4}$$

$$c = -\frac{17}{4}$$

$$c = -\frac{17}{4} \text{ or } -4\frac{1}{4} \text{ or } -4.25$$

Equations with fractions

ex. 3. $2(2y - 5) = 3(y - 6)$

$$4y - 10 = 3y - 18$$

$$4y - 3y = -18 + 10$$

$$y = -8$$

Equations with fractions

ex1. Solve for t

$$\frac{t}{2} = \frac{t}{3} + \frac{1}{2}$$

Clear fractions multiply each term in the equation by the LCM of the denominators of the fraction.

$$6 \times \frac{t}{2} = \frac{t}{3} \times 6 + \frac{1}{2} \times 6$$

LCM of 2 and 3
is 6

$$3t = 2t + 3$$

$$3t - 2t = 3$$

$$t = 3$$

ex2. Solve $\frac{1}{4}(x-2) = 5$

$$4 \times \frac{1}{4}(x-2) = 5 \times 4$$

$$1(x-2) = 20$$

$$x-2 = 20$$

$$x = 20 + 2$$

$$x = 22$$

or cross multiply

$$\frac{1}{4}(x-2) \times \frac{5}{1}$$

$$x-2 = 4 \times 5$$

$$x-2 = 20$$

$$x = 20 + 2$$

$$x = 22$$

ex3. Solve the equation

$$\frac{3x+2}{6} - \frac{2x-7}{9} = 0$$

LCM of 6 and 9
= 18

put brackets and multiply every term by 18

$$18\left(\frac{3x+2}{6}\right) - 18\left(\frac{2x-7}{-9}\right) = 18 \times 0$$

$$3(3x+2) - 2(2x-7) = 0$$

$$9x+6 - 4x+14 = 0$$

$$9x-4x+6+14 = 0$$

$$\Rightarrow 5x+20 = 0$$

$$5x+20 - 20 = 0 - 20$$

$$\frac{5x}{5} = \frac{-20}{5}$$

$$x = -4$$

ex. 4. Solve for d.

$$\frac{2d+7}{6} + \frac{d-5}{3} = \frac{d}{2} \quad \text{Lcm} = 6$$

$$6\left(\frac{2d+7}{6}\right) + 6\left(\frac{d-5}{3}\right) = 6\left(\frac{d}{2}\right)$$

$$2d+7 + 2d-10 = 3d$$

$$4d-3 = 3d$$

$$4d-3d = 3$$

$$d = 3$$

ACTIVITY

Solve for the unknowns.

(a) $3-5x=13$ (b) $4x+3=6x-4$

2(a) $\frac{1}{x} - \frac{1}{2} = 5$ (b) $\frac{3a+2}{5} - \frac{2a+3}{3} = 3$

(c) $4b - (3-b) = 17$ (d) $3(5c-1) = 4(3c+2)$

(e) $2 = 5(5y-2) - 9(3y-2)$

(f) $\frac{2m-1}{3} + \frac{3-m}{2} = \frac{m}{4}$ (g) $\frac{6a+3}{7} = \frac{2a-1}{3}$

(h) $\frac{2(5x-3)}{3} - \frac{3(5x-2)}{5} = \frac{x+2}{15}$

Equations with roots

ex 11 Solve for x

$$\sqrt{2x-5} = 3$$

we square both sides to remove the square root sign from the expression (LHS)

$$(\sqrt{2x-5})^2 = 3^2$$

$$2x-5 = 9$$

$$2x = 9+5$$

$$\frac{2x}{2} = \frac{14}{2}$$

$$x = 7$$

Note when an expression or a number with a square root sign is squared the expression or the number in the root is got.

e.g $\sqrt{2}$
 $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2}$
 $= \sqrt{4}$
 $= 2$

checking

$$\sqrt{2x-5} = 3$$

$$\sqrt{2 \times 7 - 5} = 3$$

$$\sqrt{14-5} = 3$$

$$\sqrt{9} = 3$$

$$3 = 3$$

e.g $\sqrt{2x-5}$
 $(\sqrt{2x-5})^2$
 $= (\sqrt{2x-5})(\sqrt{2x-5})$
 $= \sqrt{(2x-5)^2}$
 $= 2x-5$

ex 2. $\frac{1}{4}d = \sqrt{2}$

square both sides

$$\left(\frac{1}{4}d\right)^2 = (\sqrt{2})^2$$

$$\frac{1}{16}d^2 = 2$$

$$d^2 = 2 \times 16$$

$$\sqrt{d^2} = \sqrt{32}$$

$$d = \pm 5.657$$

or $\frac{1}{4}d = \sqrt{2}$

$$d = 4\sqrt{2}$$

$$(d)^2 = (4\sqrt{2})^2$$

$$d^2 = 16 \times 2$$

$$\sqrt{d^2} = \sqrt{32}$$

$$d = \pm 5.657$$

NOTE: Every number has two square roots
a positive and a negative

$$\text{ex. } \sqrt{4} = \pm 2 \quad \begin{array}{l} 2 \times 2 = 4 \\ -2 \times -2 = 4 \end{array}$$

There are cases where we only consider the positive square roots:

length of a line
bases.

There are no negative lengths and bases.

e.g. Solve for x given that

$$\sqrt[3]{x} = 15$$

Cube both sides to remove the cube root sign

$$\left(\sqrt[3]{x}\right)^3 = (15)^3$$

$$x = 15 \times 15 \times 15$$

$$\therefore x = 3375$$

$$\begin{array}{l} \text{Side work} \\ \left(\sqrt[3]{x}\right)^3 \\ = \left(\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x}\right) \end{array}$$

$$= \sqrt[3]{x^3}$$

$$= x$$

NOTE:

- To remove a square root sign we square both sides
- To remove a cube root sign we cube both sides
- To remove a fourth root sign we raise or power both sides to 4 etc.

ACTIVITY

1. Solve for the unknowns

$$(a) \sqrt{5x+1} = 4 \quad (b) 3 = \frac{1}{2} \sqrt[3]{3m} \quad (c) 8y^3 = 216$$

$$(d) 6a^2 = 4a^2 + 50$$

Equations from word problems

ex. 1 In a test Mark got twice as many marks as Melissa and Osteen got 8 more than Mark. Together they got a total of 223 marks. What was Melissa's mark?

Melissa	Mark	Osteen	Total
x	$2x$	$2x+8$	223

$$x + 2x + 2x + 8 = 223$$

$$5x + 8 = 223$$

$$5x = 223 - 8$$

$$\frac{5x}{5} = \frac{215}{5}$$

$$x = 43$$

∴ Melissa's mark was 43.

ex 2. Jesse is 11 years older than Jane. In 5 years' time Jesse will be thrice as old as Jane. Determine their present ages.

	Today (present age)	5 years' time
Jane	m yrs	$(m+5)$ yrs
Jesse	$(m+11)$ yrs	$(m+11)+5$ yrs / $3(m+5)$

$$\text{So } (m+11)+5 = 3(m+5)$$

$$m+16 = 3m+15$$

$$m-3m = 15-16$$

$$\frac{-2m}{-2} = \frac{-1}{-2}$$

$$m = \frac{1}{2}$$

∴ Jane is now $\frac{1}{2}$ year

$$\begin{aligned} \text{Jesse is now } (m+11) &= 11 + \frac{1}{2} \\ &= 11\frac{1}{2} \text{ years old.} \end{aligned}$$

ex.3 I think of a number, double it and add 17. The answer is 49. Look for the number

Let the number be t : $2t + 17 = 49$

$$2t + 17 = 49 \quad \leftarrow$$

$$2t = 49 - 17$$

$$\frac{2t}{2} = \frac{32}{2}$$

$$t = 16$$

$$\text{or } t + t + 17 = 49$$

$$2t + 17 = 49$$

$$2t + 17 - 17 = 49 - 17$$

$$\frac{2t}{2} = \frac{32}{2}$$

So the number is 16.

$$t = 16$$

ACTIVITY:

- I have twice as much money as my sister. Together we have Sh. 6000. How much money does my sister have?
- x represents a certain number. When the number is multiplied by 3 the result is the same as that of adding 34 to the number. Find x .
- A rectangle is 3 times as long as it is wide. If its perimeter is 56 cm. Work out its width.
- A woman is 5 times as old as her daughter. 6 years ago the sum of their ages was 36. Calculate the age of the woman.
- One stick is 9 cm longer than another. $\frac{2}{5}$ of the longer stick is equal to $\frac{1}{2}$ of the shorter stick. Determine the length of the longer stick.