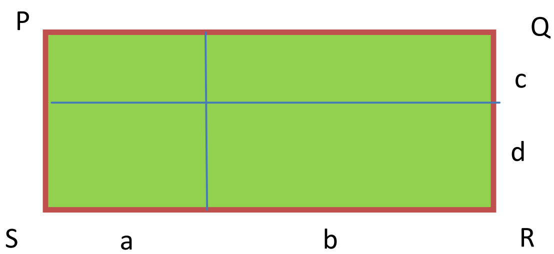
S.2 Mathematics | Binomial Products

A binomial is an expression with two terms. E.g. a + b, 2x + 3, 2 + a, etc.
An expression with two or more terms may be written with brackets as (a + b), (2x + 3), (2 + a).

Recall a(b + c) = ab + ac
The factor multiplies the two terms in the brackets.

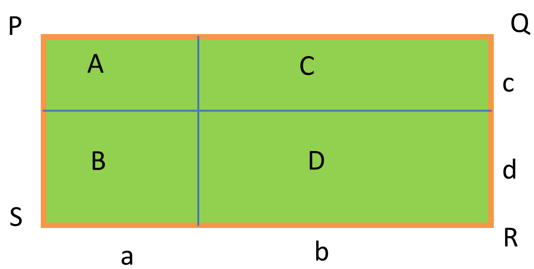
What about (a + b) (c + d)?

The figure below illustrates (a + b) (c + d).



Area of rectangle PQRS in (a + b) (c + d)……………………….**1**

But PQRS may be divided into rectangles A, B, C and D



Area A = a x c = ac
Area B = a x d = ad
Area c = b x c = bc
Area D = b x d = bd

Therefore Area of PQRS = ac + ad + bc + bd………………… **2**

From 1 and 2 above,
(a + b) (c + d) = ac + ad + bc + bd…………………………………..**3**

Note that, when we expand the above expression, each of the terms in the first pair of brackets multiplies the terms in the second pair of brackets.

i.e

(**a+b**) (c + d) = **a**(c + d) **+ b**(c + d)

= **a**c + **a**d + **b**c + **b**d

Therefore (a + b) (c + d) = ac + ad + bc + bd

When the first pair of brackets is removed, the second term goes with its sign (check **+b** above). If **b** had been **negative**, it would carry its **negative** sign.

Example 1:

Expand i) (p + q) (r + 2s)

ii) (w – 2x) (3y + z)

iii) (2e + 3f) (4f – g)

Solution

i) (**p + q**) (r + 2s) = **p**(r + 2s) **+ q**(r + 2s)

= pr + 2**p**s + **q**r + 2**q**s

ii) (w **– 2x**) (3y + z) = w(3y + z) **– 2x**(3y + z)

= 3wy + wz – 6xy + 2xz

iii) (2e + 3f) (4f - g) = 2e(4f - g) + 3f(4f - g)

=8ef – 2eg + 12f^2 – 3fg

There are no like terms in each of the expanded expressions in (i), (ii) and (iii). So the expressions cannot be simplified.

Example 2

Expand and simplify

i) (2p – q) (2p + q)

ii) (x + 2y) (2x – y) + 3x(x - y)

Solution

i) (2p - q) (2p + q) = 2p(2p + q) – q(2p + q)

= 4p^2 + 2pq – 2pq - q^2 (But 2pq – 2pq = 0)

= 4p^2 - q^2

ii) (x + 2y) (2x - y) + 3x(x - y)

= x(2x - y) + 2y(2x – y) + 3x(x – y)

= 2x^2- xy + 4xy – 2y^2 + 3x^2 – 3xy

= 2x^2 + 3x^2**+ 4xy – xy – 3xy** – 2y^2

$=5x^2 - 2y^2$

EXERCISE

1) Expand the products

a) (n + 2p) (2n + 3)

b) (2t + u) (v + 3w)

c) (3e + 2f) (g - h)

d) (2w - x) (3x + 2y)

e) (4p - q) (p + 2)

f) (2u – 3v) (v + 4w)

2) Expand the products and then simplify them by collecting like terms.

a) (a + 2b) (2a + b)

b) (e + 4f) (2e – 5f)

c) (3h - 4) (4 + 3h)

d) $(2x + y)^2$

e) $(2x - y)^2$

f) (2x + 3) (3x – 4) + (6 - x)

3) Expand the products, then simplify them by collecting like terms.

a) $(a + b)^2$ b) $(2x + 3)^2$

c) $(x - y)^2$ d) $(2a - b)^2$

e) (a + b) (a – b) f) (2x – y) (2x + y)

The three Identities

Square of the sum of two terms

1) $(a + b)^2 = (a + b)(a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

Therefore $(a + b) = a^2 + 2ab + b^2$……………………………….(i)

Square of the difference of two terms

2) $(a - b)^2 = (a - b)(a - b)$

$= a(a - b) - b(a - b)$

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

Therefore $(a - b)^2 = a^2 - 2ab + b^2$………………………………(ii)

Difference of two squares

3) (a + b) (a - b) = a(a - b) + b(a - b)

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

Therefore $(a + b)(a - b) = a^2 - b^2$……………………………….(iiii)

Example 1:

Select an appropriate identity and use it to evaluate;

i) 51^2 ii) $(13\frac{1}{2})^2$ iii) 49^2

iv) 2.95^2 v) 9 x 11

Solutions

i) Let 51 be = 50 + 1

$51^2 = (50 + 1)^2$

Use $(a + b)^2 = a^2 + 2ab + b^2$

Where a = 50 and b = 1

$(50 + 1)^2 = 50^2 + 2(50)(1) + 1^2$

$= 2500 + 100 + 1$

$= 2601$

ii) $(13\frac{1}{2})^2 = (13 + \frac{1}{2})^2$

$(a + b)^2 = a^2 + 2ab + b^2$

where a = 13 and b = $\frac{1}{2}$

$(13 + \frac{1}{2})^2 = 13^2 + 2(13)(\frac{1}{2}) + (\frac{1}{2})^2$

$= 169 + 13 + \frac{1}{4}$

$= 182\frac{1}{2}$

iii) $49^2 = (50 - 1)^2$

$(a - b) = a^2 - 2ab + b^2$

where a = 50 and b = 1

$(50 - 1)^2 = 50^2 - 2(50)(1) + 1^2$

$= 2500 - 100 + 1$

$= 2401$

iv) $2.95^2 = (3 - 0.05)^2$

$(a - b) = a^2 - 2ab + b^2$

where a = 3 and b = 0.05

$(3 - 0.05)^2 = 3^2 - 2(3)(0.05) + (0.05)$

$= 9 - 0.3 + 0.0025$

$= 8.7025$

v) 9 x 11

$(a - b)(a + b) = a^2 - b^2$

$20 = 2a \qquad 2b = 2$

$a = 10 \qquad b = 1$

$(a - b) + (a + b) = 11 + 9$

$2a = 20$

$a = 10$

$(a + b) - (a - b) = 11 - 9$

$2b = 2$

$b = 1$

Therefore $(10 - 1)(10 + 1) = 10^2 - 1^2$

$= 100 - 1$

$= 99$

Example 2:

Choose the appropriate identity and use it to expand the following.

a)