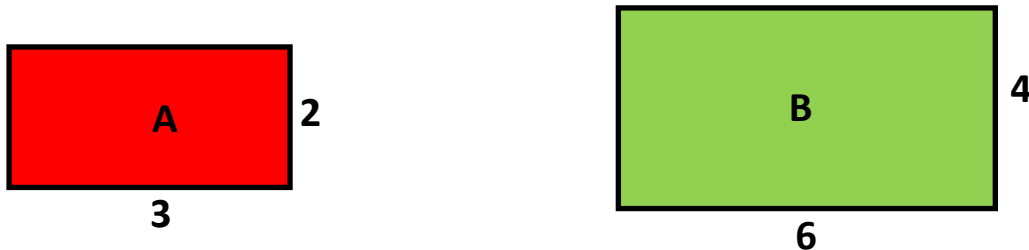


LINEAR SCALE FACTOR, AREA SCALE FACTOR, AND VOLUME SCALE FACTOR.

Linear scale factor (L.S.F)

Two plane figures or two solids are called similar if all corresponding angles are equal and if the ratio of any two corresponding lengths is constant.

The constant is called the linear scale factor. The linear scale factor is the ratio of any two corresponding lengths of similar figures.



A and B are two similar figures. The lengths of sides of figure B are twice the length of the corresponding sides of figure A.

$$\therefore \text{L.S.F} = \frac{4}{2} = \frac{6}{3} = 2$$

Area Scale factor (A.S.F)

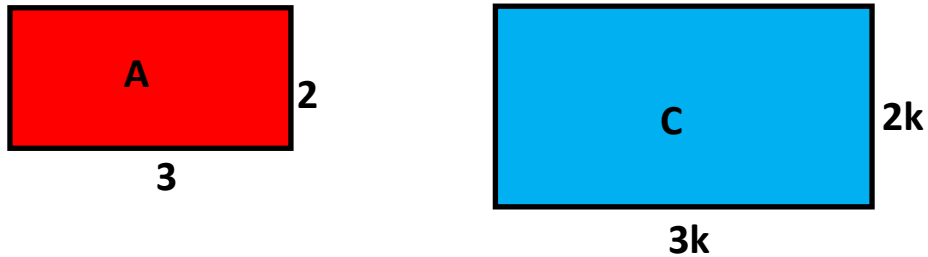
Area of A = $2 \times 3 = 6$ square units

Area of B = $6 \times 4 = 24$ square units

Ratio of their areas = $24:6 = 2^2$

This is linear scale factor squared.

Let us consider rectangle A and similar rectangle C.



$$\text{L.S.F} = \frac{3k}{3} = \frac{k}{1} = k$$

$$\text{Area scale factor} = \frac{3k \times 2k}{3 \times 2} = k^2$$

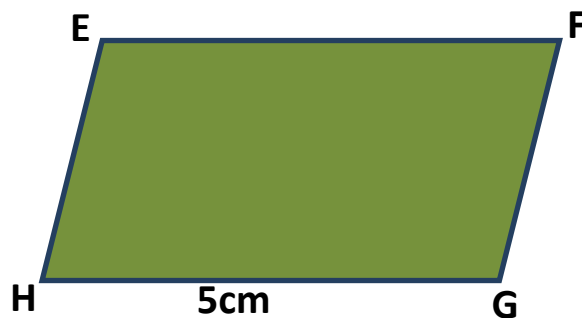
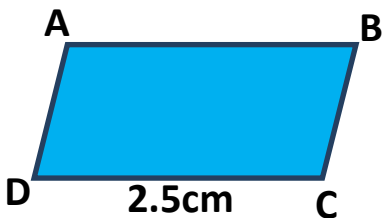
Note: if the L.S.F of two similar figures is k , then the area scale factor is k^2 .

This rule applies to all similar figures.

In each of the following three examples, the two figures are similar.

Example:1

Area of parallelogram ABCD is 5.5cm^2 . Find the area of EFGH



Solution:

$$\text{L.S.F} = \frac{HG}{DC} = \frac{5}{2.5} = \frac{2}{1} = 2$$

$$\text{A.S.F} = (\text{L.S.F})^2 = (2)^2 = 4$$

Area of EFGH = 4 x Area of ABCD

$$= 4 \times 5.5$$

$$= 22\text{cm}^2$$

Example:2

A circle x has area of 12.0cm^2 . Find the area of circle y with

i) three times this radius

ii) half this radius

Solution:

$$\text{i) L.S.F} = \frac{3}{1} = 3$$

$$\text{Area scale factor} = (3)^2 = 9$$

$$\text{Area of circle Y} = 9 \times 12.0 = 108\text{cm}^2$$

$$\text{ii) L.S.F} = \frac{1/2}{1} = \frac{1}{2}$$

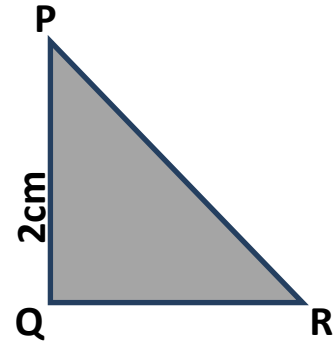
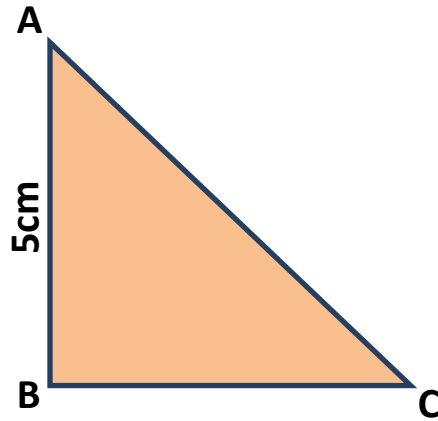
$$\text{Area scale factor is } \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Area of Y} = \frac{1}{4} \times 12 = 3\text{cm}^2$$

Example:3

Find the area of the triangle ABC, Given the area of the triangle

$$\text{ABC} = 150\text{cm}^2$$



$$\text{Area scale factor} = (\text{L.S.F})^2 = \left(\frac{2}{5}\right)^2$$

$$\begin{aligned} \text{Area of PQR} &= \left(\frac{4}{25}\right)^2 \times 150 \\ &= 24\text{cm}^2 \end{aligned}$$

Map scales

Any shape on the map is similar to the actual shape on the ground. The scale of the map is the ratio of corresponding lengths (i.e. it is the linear scale factor).

A scale factor of 1:50,000 means 1 unit on the map is equivalent to 50,000 units on the ground.

$$\text{L.S.F} \left(\frac{\text{map}}{\text{ground}} \right) = \frac{1}{50,000}$$

$$\text{Area scale factor} = \left(\frac{1}{50,000} \right)^2$$

$$\frac{\text{Area on map}}{\text{Area on ground}} = (\text{scale})^2 = \left(\frac{1}{50,000} \right)^2$$

Hence area of 1cm^2 on map represents an area of $50,000 \times 50,000\text{cm}^2$ on the ground.

$$\begin{aligned}
 1\text{cm}^2 &= 50,000 \times 50,000\text{cm}^2 \\
 &= 0.5 \times 0.5\text{km}^2 \\
 &= 0.25\text{km}^2
 \end{aligned}$$

Example:

The scale on a map is 1:100,000. If the area of an island on the map is 200cm^2

- i) What is its actual area?
- ii) What is its actual area on a map whose scale is 1:50,000?

i) Scale 1:100,000

$$\begin{aligned}
 1\text{cm}:100,000\text{cm} \\
 1\text{cm}:1\text{km}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area scale factor} &= 1\text{cm}^2 = 1\text{km}^2 \\
 200\text{cm}^2 &= 200 \times 1\text{km}^2 \\
 &= 200\text{km}^2
 \end{aligned}$$

ii) Scale = 1:50,000

$$\begin{aligned}
 1\text{cm}: 50,000 \\
 \text{Area scale factor } 1\text{cm}^2:(50,000) \\
 1\text{cm}^2=0.25\text{km}^2
 \end{aligned}$$

Let the area on the map be y

$$\begin{aligned}
 1\text{cm}^2 \text{ rep } 0.25\text{km}^2 \\
 y \text{ rep } 200\text{km}^2 \text{ (from (i) above)}
 \end{aligned}$$

Cross multiply

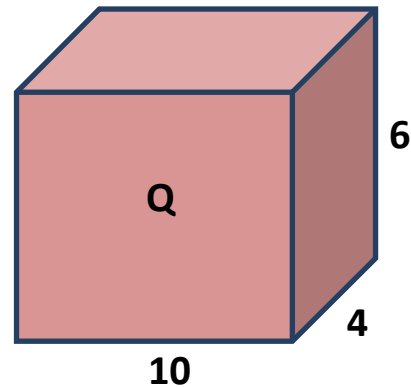
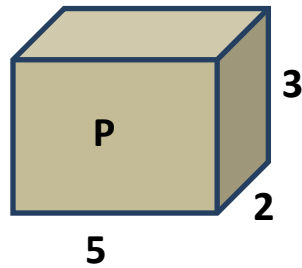
$$y \times 0.25\text{km}^2 = 1\text{cm}^2 \times 200\text{km}^2$$

$$y = \frac{1\text{cm}^2 \times 200\text{km}^2}{0.25\text{km}^2}$$

$$y = 800\text{cm}^2$$

Volume scale factor

Consider two similar cuboids P and Q.



$$\text{L.S.F} = \frac{4}{2} = \frac{2}{1} = 2$$

$$\text{A.S.F} = \frac{4 \times 6}{2 \times 3} = 2 \times 2 = (2)^2 = 4$$

This is true for each corresponding faces.

$$\begin{aligned} \text{Volume scale factor (V.S.F)} &= \frac{10 \times 4 \times 6}{5 \times 2 \times 3} \\ &= 2 \times 2 \times 2 = (2)^3 = 8 \end{aligned}$$

$$\therefore \text{V.S.F} = (\text{L.S.F})^3$$

In general, if L.S.F of two similar solids is K, then the V.S.F is K^3

Example:

Two similar cylinders are such that the height of the larger one is three times that of the smaller one. The smaller one has

surface area of 27cm^2 and volume 6cm^3 .

What are the area and volume of the larger one?

Solution:

$$\text{L.S.F} = 3$$

$$\text{A.S.F} = (3)^2 = 9$$

$$\text{V.S.F} = (3)^3 = 27$$

Area of the larger one = $9 \times$ Area of smaller one

$$\text{Area of the larger one} = 9 \times 27 = 243\text{cm}^2$$

$$\begin{aligned}\text{Volume of larger one} &= 27 \times \text{volume of small one} \\ &= 27 \times 6 = 162\text{cm}^3\end{aligned}$$

Example:

Two beakers of similar shapes hold $\frac{1}{4}$ litre and 2 litres respectively.

- i) The smaller is 4cm high. What is the height of the larger one?
- ii) If the Larger one has surface area of 400cm^2 , what is the area of the smaller one?

Solution:

i) The volumes are 2 litres and $\frac{1}{4}$ litres

$$\text{V.S.F} = \frac{2}{\frac{1}{4}} = 8 = (2)^3$$

$$\text{L.S.F} = \sqrt[3]{\text{V.S.F}} = \sqrt[3]{2^3} = 2$$

Height of the larger one is $4 \times 2 = 8\text{cm}$.

ii) A.S.F = (L.S.F)² = 2² = 4

Area Of small one = $\frac{400}{4} = 100\text{cm}^2$

Exercise:

1. A 1-litre beaker is 15cm high. Find the capacity of a 7.5cm high similar beaker.
2. Two similar rectangular cartons have bases 10cm long and 30cm. The smaller has volume 1200cm³. Find the volume of the larger one.
3. The volume of two cubes are 12.5cm³ and 100cm³. Find their
 - i) Linear ratio.
 - ii) Area ratio.
4. Two triangles whose bases are 7.5cm and 2.5cm are similar. What is the area of the smaller triangle if the larger one has area 81cm²
5. A model of an aero plane constructed to a scale of 1:600 is 56cm long. What is the length of the original aero plane?
6. Two similar solids have linear scale factor 3. If the larger one has a surface area of 36cm² and volume of 12cm³, what are the surface area and volume of the other?
7. On a map of scale 1:200 000, a town is represented by an area of 4cm². Calculate in km² the area covered by the town on ground?

8. The photograph of a house actually 8m high is 10cm in height.
- a) The front door of the house is 2.4m high. How high is that door on the photograph?
- b) If the area of a window on the photograph is 1.4cm^2 , what is the area of the actual window? (Give your answer to the nearest 0.1m^2)
9. A scale model of a ship is one two-hundredth as long as the ship itself.
- i) Find the height of the mast and the area of the deck of the ship if the height of the mast and the deck area of the model are 10cm and 900cm^2 respectively
- ii) If the volume of the hold in the ship is 1600m^3 , find the volume of the hold in mm^3 , in the model.

Stay Home

Stay Safe