

STANDARD HIGH SCHOOL ZZANA

A GUIDE TO A LEVEL PHYSICS PRACTICAL

Significant figures

The **significant figures** (also known as the **significant digits**) of a number are digits that carry meaning contributing to its measurement resolution (accuracy and precision). This includes all digits *except*:

- All leading zeros;
- Trailing zeros when they are merely placeholders to indicate the scale of the number (exact rules are explained at identifying significant figures); and
- Spurious/false digits introduced, for example, by calculations carried out to greater precision than that of the original data, or measurements reported to a greater precision than the equipment supports.

Significance arithmetic are approximate rules for roughly maintaining significance throughout a computation.

Numbers are often rounded to avoid reporting insignificant figures. For example, it would create false precision to express a measurement as 12.34500 kg (which has seven significant figures) if the scales only measured to the nearest gram and gave a reading of 12.345 kg (which has five significant figures). Numbers can also be rounded merely for simplicity rather than to indicate a given precision of measurement, for example, to make them faster to pronounce in news broadcasts.

□ Identifying significant figures

Concise rules

- All non-zero digits are significant: 1, 2, 3, 4, 5, 6, 7, 8, 9.
- Zeros between non-zero digits are significant: 102, 2005, 50009.
- Leading zeros are never significant: 0.02, 001.887, and 0.000515.
- In a number *with* a decimal point, trailing zeros (those to the right of the last non-zero digit) are significant: 2.02000, 5.400, 57.5400.
- In a number *without* a decimal point, trailing zeros may or may not be significant. More information through additional graphical symbols or explicit information on errors is needed to clarify the significance of trailing zeros.

Significant figures rules explained

Specifically, the rules for identifying significant figures when writing or interpreting numbers are as follows:

- All non-zero digits are considered significant. For example,
 - (i) 91 has two significant figures (9 and 1)
 - (ii) 123.45 has five significant figures (1, 2, 3, 4 and 5).
- Trapped zeroes: Zeroes appearing anywhere between two non-zero digits are significant: For example,
 - (i) 101.1203 has seven significant figures: 1, 0, 1, 1, 2, 0 and 3.
 - (ii) 6.0037 has five significant figures: 6,0,0,3 and 7

- Leading zeroes: Zero to the left (of the significant figures) are not significant. For example,
 - (i) 0.00052 has two significant figures: 5 and 2.
 - (ii) 0.000789 has three significant figures: 7, 8 and 9
- Trailing zeroes: Zero to the right of the significant figures in a number containing a decimal point are significant. For example,
 - (i) 12.2300 has six significant figures: 1, 2, 2, 3, 0 and 0.
 - (ii) The number 0.000122300 still has only six significant figures (the zeros before the 1 are not significant).
 - (iii) In addition, 120.00 has five significant figures since it has three trailing zeros. This convention clarifies the precision of such numbers; for example, if a measurement precise to four decimal places (0.0001) is given as 12.23 then it might be understood that only two decimal places of precision are available. Stating the result as 12.2300 makes clear that it is precise to four decimal places (in this case, six significant figures).
- The significance of trailing zeros in a number not containing a decimal point can be ambiguous. For example, it may not always be clear if a number like 1300 is precise to the nearest unit (and just happens coincidentally to be an exact multiple of a hundred) or if it is only shown to the nearest hundred due to rounding or uncertainty. Many conventions exist to address this issue:
 - An overline, sometimes also called an overbar, or less accurately, a vinculum, may be placed over the last significant figure; any trailing zeros following this are insignificant. For example, 1300 has three significant figures (and hence indicates that the number is precise to the nearest ten).
 - Less often, using a closely related convention, the last significant figure of a number may be underlined; for example, "2000" has two significant figures.
 - A decimal point may be placed after the number; for example "100." indicates specifically that three significant figures are meant.
 - In the combination of a number and a unit of measurement, the ambiguity can be avoided by choosing a suitable unit prefix. For example, the number of significant figures in a mass specified as 1300 g is ambiguous, while in a mass of 1.3 kg it is not.

Scientific notation

In most cases, the same rules apply to numbers expressed in scientific notation. However, in the normalized form of that notation, placeholder leading and trailing digits do not occur, so all digits are significant. For example,

- (i) 0.00012 (two significant figures) becomes 1.2×10^{-4}
- (ii) 0.00122300 (six significant figures) becomes 1.22300×10^{-3} .
- (iii) In particular, the potential ambiguity about the significance of trailing zeros is eliminated. For example, 1300 to four significant figures is written as 1.300×10^3 , while 1300 to two significant figures is written as 1.3×10^3 .

To round to n significant figures:

- Identify the significant figures before rounding. These are the n consecutive digits beginning with the first non-zero digit.
- If the digit immediately to the right of the last significant figure is greater than 5 or is a 5 followed by other non-zero digits, add 1 to the last significant figure. For example, 1.2459 as the result of a calculation or measurement that only allows for 3 significant figures should be written 1.25.
- If the digit immediately to the right of the last significant figure is a 5 not followed by any other digits or followed only by zeros, rounding requires a tie-breaking rule. For example, to round 1.25 to 2 significant figures:
 - Round half away from zero (also known as "5/4")¹ rounds up to 1.3. This is the default rounding method implied in many disciplines if not specified.
 - Round half to even, which rounds to the nearest even number, rounds down to 1.2 in this case. The same strategy applied to 1.35 would instead round up to 1.4.
- Replace non-significant figures in front of the decimal point by zeros.
- Drop all the digits after the decimal point to the right of the significant figures (do not replace them with zeros).

Precision	Rounded to significant figures	Rounded to decimal places
6	12.3450	12.345000
5	12.345	12.34500
4	12.35	12.3450
3	12.3	12.345
2	12	12.35
1	10	12.3
0	N/A	12

Another example for **0.012345**:

Precision	Rounded to significant figures	Rounded to decimal places
7	0.01234500	0.0123450
6	0.0123450	0.012345
5	0.012345	0.01235
4	0.01235	0.0123
3	0.0123	0.012
2	0.012	0.01
1	0.01	0.0
0	N/A	0

Arithmetic /Rules for data manipulation

As there are rules for determining the number of significant figures in directly *measured* quantities, there are rules for determining the number of significant figures (s.f) in quantities *calculated* from these *measured* quantities:

- **A float value (Constants and whole numbers):** is a value with an infinite number of decimal places and significant figures.

Only *measured* quantities are used into the determination of the number of significant figures in *calculated quantities*. For example:

- (i) Exact mathematical quantities like the π in the formula for the area of a circle with radius r , πr^2 has no effect on the number of significant figures in the final calculated area.
- (ii) Similarly the $\frac{1}{2}$ in the formula for the kinetic energy of a mass m with velocity v , $\frac{1}{2}mv^2$, has no bearing on the number of significant figures in the final calculated kinetic energy. The constants π and $\frac{1}{2}$ are considered to have an *infinite* number of significant figures.

NB: if a whole number is a value of a varying quantity, it is not a float value because it has a precise number of significant figures s.f and decimal place d.p e.g 100g, 200g.....3s.f and 0 d.p

- **Multiplication and division of quantities:** For *calculated* quantities created from (many significant figures as the *measured* number with the *least* number of significant figures. For example:

- (i) $1.234_{(4 \text{ s.f})} \times 2.0_{(2 \text{ s.f})} = 2.468\dots$ record $2.5_{(2 \text{ s.f})}$
- (ii) $5.16 \times 10^{-3}_{(3 \text{ s.f})} \times 3.219_{(4 \text{ s.f})} = 16.61004 \times 10^{-3}$ record $16.6_{(3 \text{ s.f})} \times 10^{-3}$
- (iii) $\frac{4.045_{(4 \text{ s.f})} \times 0.0465_{(3 \text{ s.f})}}{4.2_{(2 \text{ s.f})}} = 0.0447839$ record $0.045_{(2 \text{ s.f})}$
- (iv) $(2.78)^2 = 2.78_{(3 \text{ s.f})} \times 2.78_{(3 \text{ s.f})} = 7.7284$ record $7.73_{(3 \text{ s.f})}$

- **Addition and subtraction of quantities:** For *calculated* quantities created from measured quantities by **addition** and **subtraction**, decimal places (d.p) are counted (not the significant figures). The decimal places (d.p) of the answer should be the same as the least number of decimal places in any of the numbers being added or subtracted i.e maintain the lowest number of decimal places (d.p). For example:

- (i) $100.0_{(1 \text{ d.p})} + 1.234_{(3 \text{ d.p})} = 101.234$ record $101.2_{(1 \text{ d.p})}$
- (ii) $1.973_{(3 \text{ d.p})} - 1.52_{(2 \text{ d.p})} = 0.453$ record $0.45_{(2 \text{ d.p})}$
- (iii) $78.9_{(1 \text{ d.p})} - 5.942_{(3 \text{ d.p})} + 11.81_{(2 \text{ d.p})} = 84.768$ record $84.8_{(1 \text{ d.p})}$

NB. The rules for calculating significant figures for multiplication and division are opposite to the rules for addition and subtraction. For multiplication and division, only the total number of significant figures in each of the factors matter; the decimal place of the last significant figure in each factor is irrelevant. For addition and subtraction, only the decimal place of the last significant figure in each of the terms matters; the total number of significant figures in each term is irrelevant.

- Trigonometric ratios logarithms should be expressed to 3(three) decimal places (3 d.p) eg
 - (iv) $\sin 36 = 0.588$ (3 d.p)
 - (v) $\cos 64.6 = 0.429$ (3 d.p)
 - (vi) $\tan 84.6 = 10.589$ (3 d.p)
 - (vii) $\log 32.45 = 1.511$ (3 d.p)