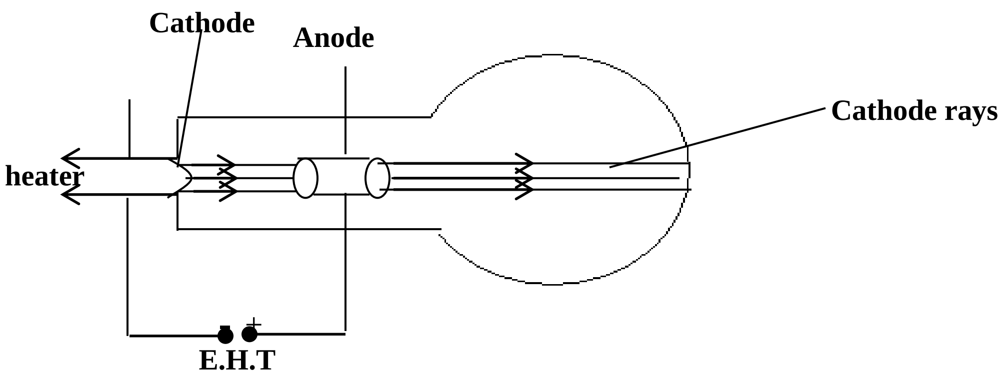
STANDARD HIGH SCHOOL ZZANA

MODERN PHYSICS

CATHODE RAYS

Cathode rays are highly energetic electrons moving from the cathode to the anode. They are produced in a cathode ray tube.

Production of cathode rays



The electrons are produced at the cathode by thermionic emission and are accelerated towards the screen by the anode which is connected to the terminal of the extra high tension battery. The thermionic emission is the process whereby metal surfaces emit electrons when heated.

The tube is evacuated to avoid electrons interacting with any particle before they reach the screen. When the cathode rays hit the florescent screen, the screen glows. This shows that electrons posses momentum and therefore have mass.

Properties of cathode rays

i. They are negatively charged

ii. They travel in a straight line

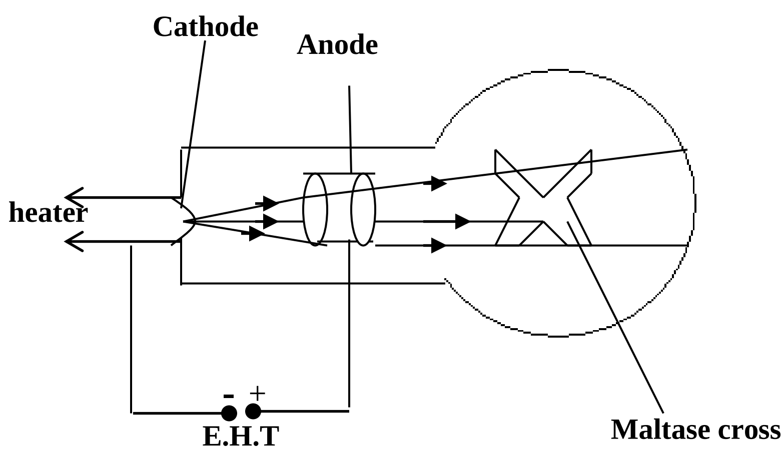
iii. They are deflected by both magnetic and electric fields (this proves that they carry a charge)

iv. They cause fluorescence in certain materials.

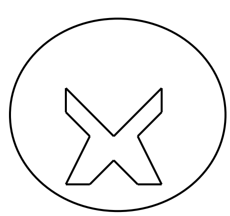
v. When cathode rays are stopped by heavy metals, x-rays are emitted.

vi. They are electrons moving with high speeds.

Verification that electrons travel in a straight line

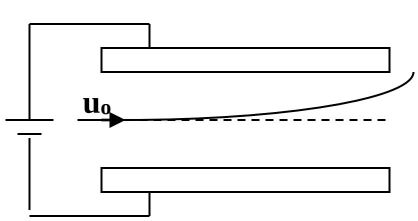


If an opaque object (Maltese cross) is placed in the path of the cathode rays, a sharp shadow of the Maltese cross is cast on the screen.



Shadow of the Maltese cross cast on the florescent screen

Motion of cathode rays in electric field



Consider an electron of speed u_o entering mid way between metal plates P_1 and P_2 separated by a distance d and across which a pd V is applied. The electric force has no component in the horizontal direction. Hence, the horizontal component of the electron velocity stays constant at the values u_o. The vertical component of the electrons velocity at a time t, after the electron has entered the region of electric field is $V_y = a_y t$

But electric force in the vertical direction

$$F = Ee$$

$$ma_y = Ee$$

$$a_y = \frac{Ee}{m}$$

$$V_y = \frac{Ee}{m}t ------(1)$$

The vertical displacement of the electron in the electric field

$$y = u_y + \tfrac{1}{2} a_y t^2$$

$$u_y = 0\ ,\ \ \mathrm{a_y} = \frac{\mathrm{Ee}}{\mathrm{m}}$$

$$y = \frac{1}{2}\frac{Ee}{m}t^2 --------(2)$$

Horizontal displacement

$$X = u_o t$$

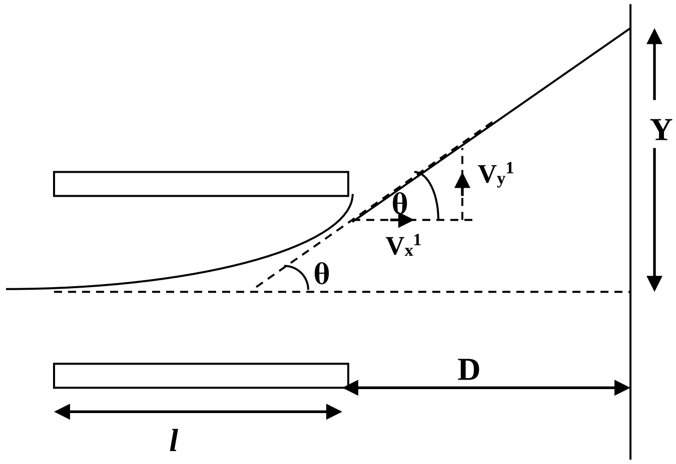
$$t = \frac{x}{u_o} ------(3)$$

Put equation(3) in equation (2)

$$y = \frac{1}{2}\frac{Ee}{m}\left(\frac{x}{u}\right)^2$$

$$y = \frac{Eex^2}{2mu_o^2}$$

The motion of the electron in the field is parabolic



The time taken by the electron to move through the plates $t_1 = \frac{l}{u_0}$

The vertical component of velocity as the electron emerges out of the field is

$$V_y^1 = \frac{Eet}{m} = \frac{Eel}{mu_0}$$

The horizontal component of velocity as the electron emerges out of the field

$$V_x^1 = u_o$$

The angle θ, the electron makes with the horizontal as it emerges out of the field is given by

$$\tan\theta = \frac{V_y^1}{V_x^1} = \frac{Eel}{Mu_o} \times \frac{1}{u_o}$$

$$\tan\theta = \frac{Eel}{mu_o^2}$$

But tan θ = $\frac{Y}{\left(D + \frac{1}{2}l\right)}$

Hence $\frac{Eel}{mu_o^2} = \frac{y}{\left(D + \frac{1}{2}l\right)}$

The vertical displacement on the screen,

$$Y = \frac{\left(D + \frac{1}{2}l\right)Eel}{muo^2}$$
$$= \frac{\left(D + \frac{1}{2}l\right)Eel}{2K}$$

Where k is the kinetic energy

Note: When an electron is accelerated by a pd of V_s volts, then the kinetic energy of the electrons is given by $\frac{1}{2}mu_o^2 = eV_s$

A beam of electrons of speed 1 x 10^6 ms^{-1} is directed midway between p and Q at right angles to the electric field between p and Q. Show that the electron emerges from the space between p and Q at an angle of 64.6^0 to the initial direction of the beam.

$$\tan\theta = \frac{v_y}{v_x}$$
$$\tan\theta = \frac{Eel}{mu_o^2}$$
$$m = 9.11\times10^{-31}kg$$
$$e = 1.6\times101^{-19}C$$
$$E = \frac{v}{d} = \frac{12}{4\times10^{-2}} = 300Vm^{-1}$$
$$\tan\theta = \frac{300\times1.6\times10^{-19}\times0.04}{9.11\times10^{-31}\times\left(1\times10^{6}\right)^2}$$
$$= \frac{1.92\times10^{-18}}{9.11\times10^{-19}} = 2.017$$
$$\tan\theta = 64.6^\circ$$

Exercise

1. An electron operating at 3 x 10^3 V is used to project electron into the space between two oppositely charged parallel plates of length 10cm and reparation 5cm. calculate the deflection of the electron as they emerge from the region between charged plated when the p.d is 1 x 10^3V.

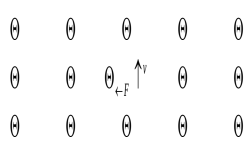
2. An electron of energy 10KeV enter midway between two horizontal metal plates each of length 5.0cm separated by a distance of 2cm. A p.d of 20V is

applied across the plates. A fluorescent screen is placed 20cm beyond the plates.

Calculate the vertical deflection on the screen.

Motion of electron in magnetic field

Consider an electron projected with a speed V at right angle to a uniform magnetic field of flux density B, out of the plane of the page.



The magnetic force on the electron is F = $Be\upsilon$ and using Flemings left hand rule it is at right angles to both u and B.

The rate of change of kinetic energy of the electron is equal to the force x velocity

$\frac{dk}{dt} = \vec{F}.\vec{\upsilon}$ Where k= kinetic energy

But since $\vec{F}$ is perpendicular to υ

Then $\frac{dk}{dt} = \vec{F}.\vec{\upsilon} = 0$

Hence the kinetic energy K = constant.

Therefore the speed v is constant

This implies that the electron moves in a circular orbit.

$$\frac{m\upsilon^2}{r} = Be\upsilon$$

$$r = \frac{m\upsilon}{Be}$$

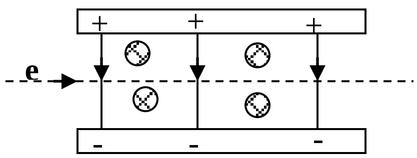
The period T = $\frac{2\pi}{\omega} = \frac{2\pi m}{Be}$

The kinetic energy

$$k = \frac{1}{2} m\upsilon^2 = \frac{1}{2} m\left(\frac{Ber}{m}\right)^2$$

$$= \frac{1}{2}\frac{B^2e^2r^2}{m} = \frac{B^2e^2r^2}{2m}$$

Motion of electron in crossed field (magnetic and electric)



Consider an electron projected with a speed v at right angles to both electric intensity E and magnetic flux density B.

The electric force on an electro F_e = Ee downwards. The magnetic force on an electron

F_m = Bev upwards. If the electron passes through the crossed fields undeflected

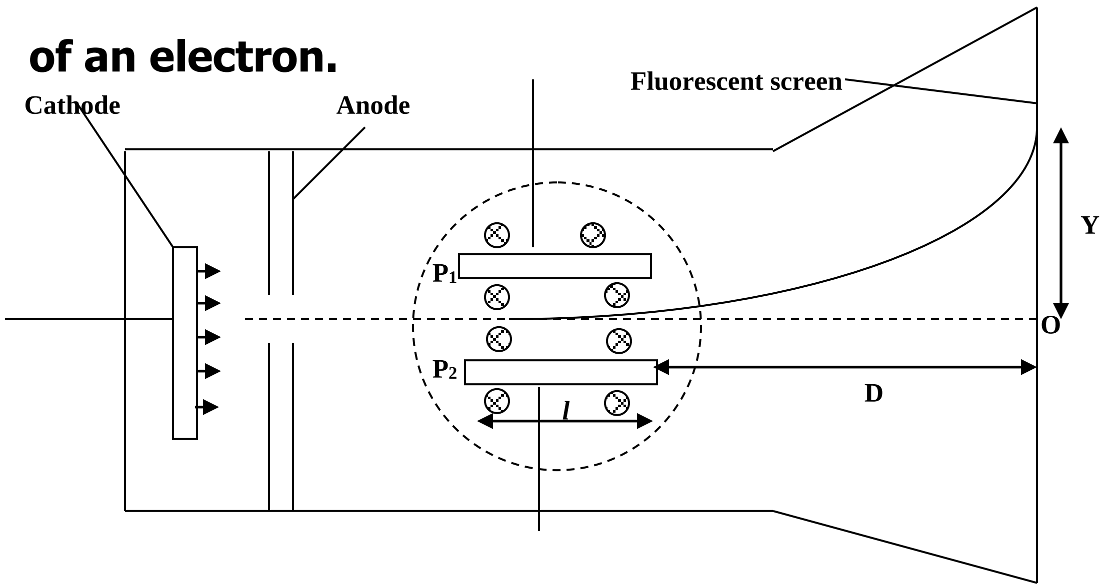
then $F_e = F_m$.

Ee = Bev

$$E = Bv$$

$$v = \frac{E}{B} \text{ as long as electrons are not deflected}$$

Thomson's method used to measure the charge to mass ration ($\frac{e}{m}$) of an electron.



Electron emitted by the heated cathode is accelerated to the anode. In the absence of the electric and magnetic fields, the cathode rays strike the florescent screen at O. The p.d is then applied between plate's P_1 and P_2. The deflection Y of the cathode rays is measured. A magnetic field is then applied to the plane of the figure by passing current through a pair of circular coil one on each side of the glass tube.

The current in the circular coil is varied until the fluorescent spot is brought back to O. When cathode rays are in the electric field alone,

$$\frac{e}{m} = \frac{Yu_o^2}{El\left(D + \frac{1}{2}l\right)}$$

When the magnetic field is applied such that the electron beam is brought at O, then

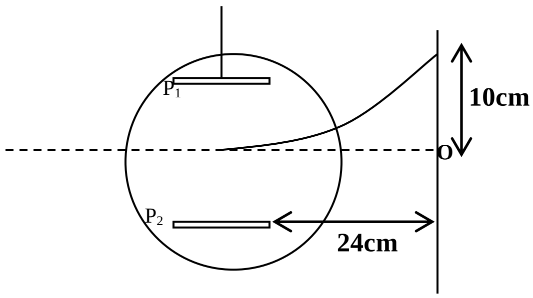
$$Beu_o = Ee$$

$$u_o = \frac{E}{B}$$

$$\frac{e}{m} = \frac{YE}{B^2 l\left(D + \frac{1}{2}l\right)}$$

But $E = \frac{V}{d}$ where d is the separation of plates P_1 and P_2

Example



In the figure, p_1 and p_2 are metal plates each of length 2cm and separated by 0.5cm in a uniform magnetic field of flux density 4.7×10^{-3}T.

An electron beam incident midway between the plates is deflected by the magnetic field by a distance of 10cm on a screen placed a distance of 24cm from the ends of the plates. When a p.d of 10^3V is applied between P_1 and P_2, the electron spot on the screen is restored to the undeflected path O, calculate the e/m of the electron.

$$\frac{e}{m} = \frac{YE}{B^2 l\left(D + \frac{1}{2}l\right)}$$

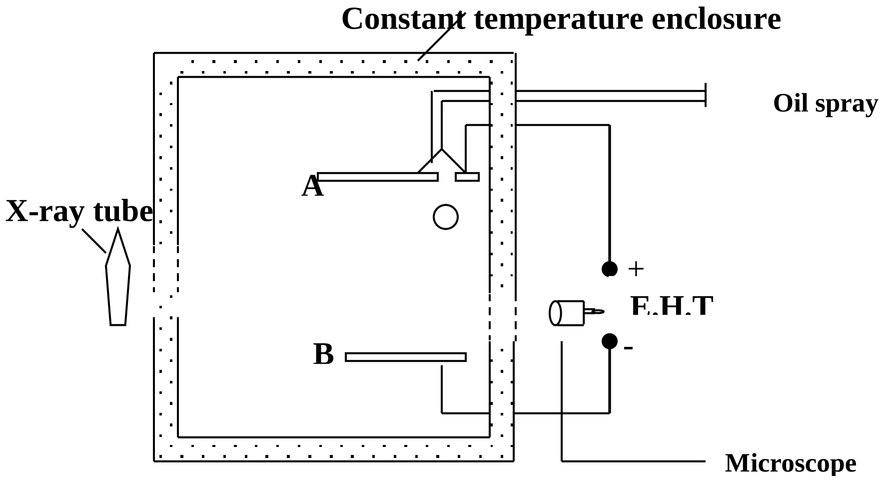
$$E = \frac{V}{d} = \frac{10^3}{0.5 \times 10^{-2}}$$

$$\frac{e}{m} = \frac{10 \times 10^{-2} \times 2.0 \times 10^5}{\left(4.7 \times 10^{-3}\right)^2 \times 2 \times 10^{-2} \times (0.24 + 0.01)}$$

$$= \frac{2 \times 10^4}{1.1045 \times 10^{-7}}$$

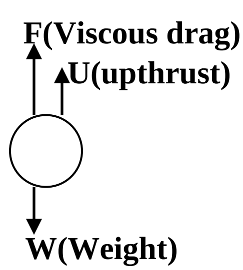
$$= 1.81 \times 10^{11}\,Ckg^{-1}$$

Measurement of electric charge using Milkan's oil drop experiment



Two horizontal plates A and B are connected to an extra high tension battery E.H.T such that a vertical field is created between the plates. The apparatus is surrounded by a liquid bath to provide constant temperature, an oil spray is introduced into the field through a vent in the upper plate. The oil droplets acquire some charge by friction. Additional charge on the drops can be provided by radiating the region between the plates with X rays. X rays cause ionisation of atoms they interact with. The chamber is illuminated with intense light and the oil drop observed through a short focus travelling microscope. The drops are allowed to drop freely when the electric field is put off. The terminal velocity V_o of the drop is measured by measuring the distance it falls through in a measured time.

During free fall



$F = 6\pi a\eta v_o$

$W = \frac{4\pi a^3}{3}\rho g$

$U = \frac{4\pi a^3}{3}\sigma g$

where ρ and σ are the drop and air densities respectively and a is radius of drop.

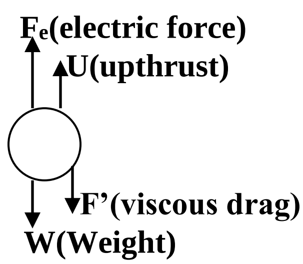
At terminal velocity, $W = U + F$..................(1)

$$\frac{4}{3}\pi a^3\rho g = \frac{4}{3}\pi a^3\sigma g + 6\pi a\eta v_o$$

Hence $a^2 = \frac{9\eta v_0}{2g(\rho - \sigma)}$

Therefore determining terminal velocity of fee fall, the radius a can be got.

Case 1 When the p.d is applied such that the oil drop rises steadily,



At terminal velocity , v_1

$F_e + U = W + F'$............(2)

Put equation (1) into (2)

$F_e + U = U + F + F'$

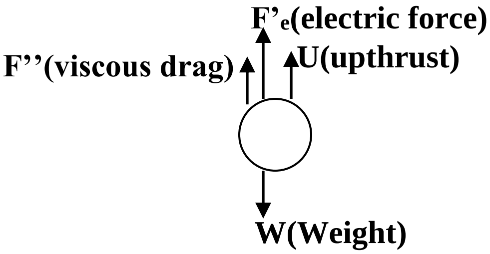
$F_e = F + F'$

$Eq = 6\pi a\eta v_o + 6\pi a\eta v_1$

$q = \frac{6\pi a\eta(v_0 + v_1)}{E}$

But $E = \frac{V}{d}$ where d is separation of plates

Case2 When the p.d V is applied such that the drop falls steadily with a speed v_2.

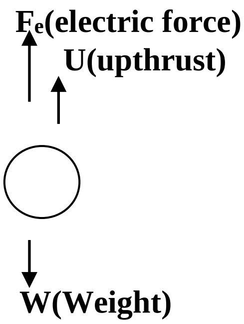


At terminal velocity, $W = F_e^{'} + U + F^{''}$ ....(3)

Sub Eqn (1) in eqn (3)

$$F_e^{'} = F - F^{''}$$
$$Eq = 6\pi a\eta v_0 - 6\pi a v_2$$
$$q = \frac{6\pi a\eta d}{V}(v_0 - v_2)$$

Case3 When the p.d is applied such that the drop remains stationary



$W = U + F_e$..................(4)

sub. Eqn (1) in (4)

F = F_e

$$6\pi a\eta v_0 = Eq$$
$$q = \frac{6\pi a\eta d v_0}{V}$$

After repeating the experiment for different oil drops, Milkan found out that the charges on the drop were integer multiples of the value 1.6×10^{-19}C. This value was taken to be the basic charge and is the charge on the drop.

Exercise

1. An oil drop carrying a charge of 3e falls under gravity in air with a velocity $4.6x10^{-4}$ ms^{-1} between two parallel plates mm apart. When a p.d of $4.6x10^{3}V$ is applied between the plates, the drop rises steadily assuming the effect of the air buoyancy on the drop is negligible. Calculate

   (i) the radius of the drop ($2.06x10^{-6}m$)

   (ii) the velocity with which the oil drop rises

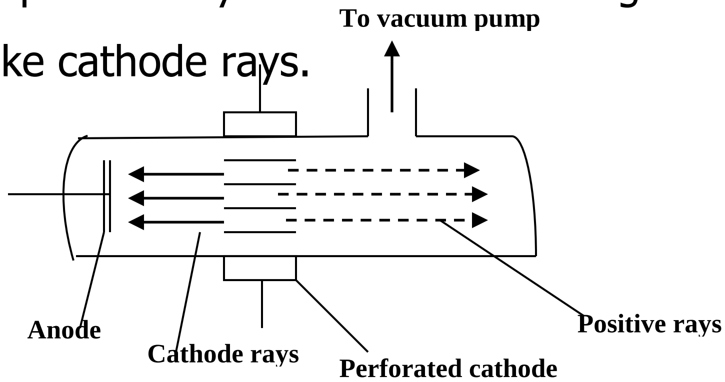
   (density of oil =$900kgm^{-3}$, viscosity of air = $1.8x10^{-5}Nsm^{-2}$)

2. A charged oil drop of radius $7.26x10^{-7}m$ and density $880kgm^{-3}$ is held stationary in an electric field of intensity $1.72x104Vm^{-1}$. How many charges on the drop (density of air = $1.29kgm^{-3}$)

Positive rays

At low pressures, in a discharge tube, electrons from the perforated cathode ionise gas atoms in the tube. The positive ions produced and accelerated to high energies are called positive rays.

The positive rays are related to the gas atoms initially in the discharge tube unlike cathode rays.



Properties of Positive rays

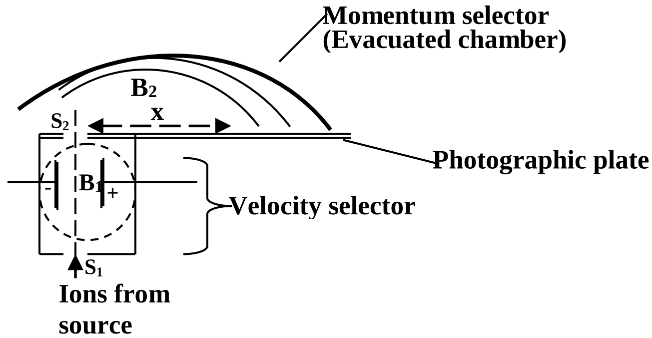
They are deflected by the electric and magnetic fields to a lesser extend than cathode rays. This is because they are more massive than cathode rays.

They are deflected by the electric and magnetic fields in a direction opposite those in which cathode rays are deflected. This because they are positively charged.

They exhibit a range of speeds because they are produced at various points along the discharge tube between the cathode and the anode.

They cause fluorescent and affect photographic plates such as cathode rays.

Measurement of specific charge of ions using Bain Bridge Mass Spectrometer.



Ions from the source enter the velocity selector through collimating slit s_1. The ions enter the velocity selector and only ions with velocity $v = \frac{E}{B_1}$ pass through undeflected. E is the electric field intensity of electric field in the momentum selector, B_1 is the magnetic flux density of the magnetic field in the velocity selector.

When these ions reach the momentum chamber, they are deflected by the magnetic field of magnetic flux density B_2 and describe a circular arc and strike the photographic plate.

Hence in the momentum chamber, $\frac{mv^2}{r} = B_2 qv$

Hence $\frac{q}{m} = \frac{v}{B_2 r}$, But $r = \frac{x}{2}$

Where x is the distance between a point on the photographic plate where the ion strike and slit s_2.

Also, $v = \frac{E}{B_1}$

Hence $\frac{q}{m} = \frac{2E}{B_1 B_2 x}$

For two ions of masses m_1 and m_2;

$\frac{q}{m_1} = \frac{2E}{B_1 B_2 x_1}$, $\frac{q}{m_2} = \frac{2E}{B_1 B_2 x_2}$

Hence the separation of ions on the plate

$$x_2 - x_1 = \frac{2E}{qB_1B_2}(m_2 - m_1)$$

Example

In a mass spectrum, two ions of mass 26,28 with charges +10e and +30e respectively. Both enter magnetic fields B2 with the same velocity. The radius of a circular path described by a heavier ion 0.28m, find the separation of two images formed on a photographic plate by these ions.

From $\frac{q}{m} = \frac{2E}{B_1B_2x}$

Hence $\frac{q}{m}\alpha x$, since $\frac{2E}{B_1B_2}$ is constant

But $\frac{q_1}{m_1} = kx_1$, $\frac{q_2}{m_2} = kx_2$

Therefore $\frac{q_1m_2}{q_2m_1} = \frac{x_1}{x_2}$

But $m_1 = 26, m_2 = 28, q_1 = 10e, q_2 = 30e, x_1 = 2\times 0.28m = 0.56m$

$$\frac{10e\times 28}{30e\times 26} = \frac{0.56}{x_2}$$

$$x_2 = 1.56m$$

$$separation = x_2 - x_1 = 1.56 - 0.56 = 1.00m$$

Exercise

1. Ionised gas atoms produce in a discharge tube. They enter slits S1 of the Bain Bridge spectrometer. The charges pass through B_1 of 0.5T applied at 90 to the electric fields (3V, 5cm). The ions then pass through undeviated. The beam next passes through B_2 of 0.8T and moves along the two circular path. The radius of the small path is 18cm. The separation on the plates is 6cm. If the charge on the ions is 10C, identify the ions (m_1=12g, m_2 = 14g, hence ion is carbon).

2. A stream of singly ionised magnesium atoms is accelerated through a p.d of 50V, and then enters a region of uniform magnetic field of flux density 2.08×10^{-2}T. calculate the atomic mass of the ions. (24U or 3.987×10^{-26}kg)

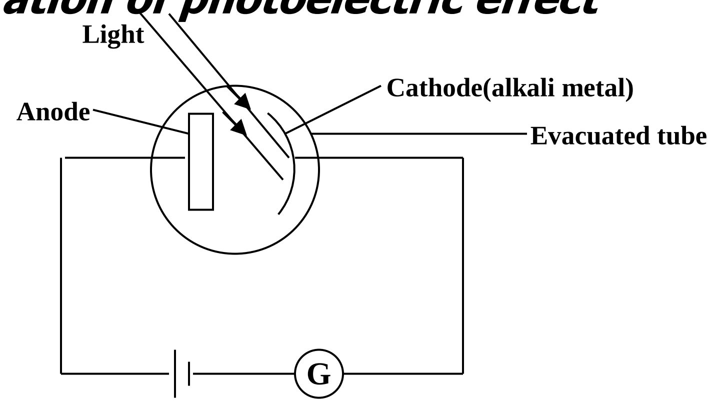
3. The mass of the singly charged neon isotope ${}^{20}_{10}Ne^{+}$ is 3.3×10^{-26}kg. A beam of these ions enters a uniform transverse magnetic field of mass 0.3T, and describes a circular orbit of radius 0.22m. What is (i) the velocity of the ions

(ii) the p.d which has been used to accelerate them to this velocity?

PHOTOELECTRIC EFFECT

When some metals held at a negative potential are illuminated by electromagnetic radiations, electrons are emitted. This process is called *photoelectric emission*.

Demonstration of photoelectric effect

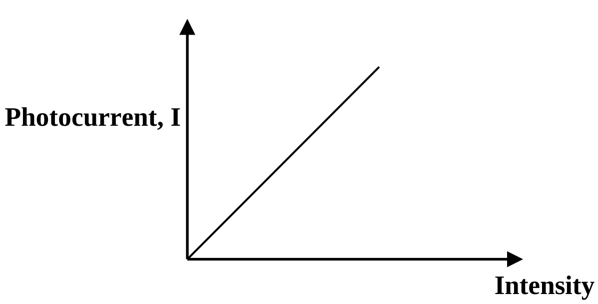


When light falls on a metal cathode, a galvanometer shows a deflection, indicating flow of current. However when the plates are covered, more current flows. Energy of the incident light is absorbed by the electrons and instantly an electron jumps out. Such ejected electrons are called photoelectrons.

Experimental observations on photoelectric effect.

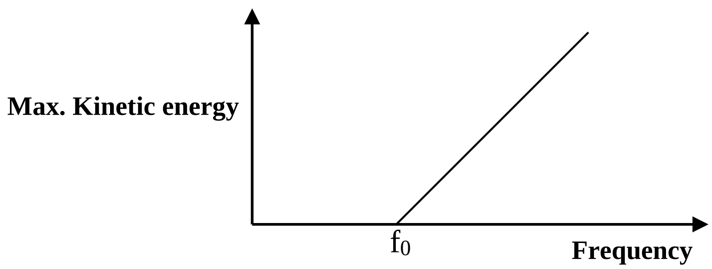
1. There is negligible time delay between irradiation of metal surface and emission of electrons by the surface.

2. The photocurrent(number of photoelectrons per second) is proportional to the intensity of the incident radiation.



3. The maximum kinetic energy of photo electrons increases linearly with the frequency of the incident radiation but is independent of the intensity of the radiation.

4. For each metal surface, there is a minimum frequency, f_0 of the incident radiation below which no electrons are emitted however high is the intensity. This frequency is called *Threshold frequency* of the metal surface.



Classical theory and photoelectric effect

According to the classical theory of radiation, radiant energy is evenly distributed in a wave front. If the radiation is incident on a metal surface, the surface electrons should share equally the energy delivered to the surface by the radiation. Photoelectric emission should occur when electrons have gained sufficient energy from the radiation. The order of magnitude of the delay time predicted by the classical theory is approximately 10^3 seconds which contradicts with the observed 10^{-9} second. Hence the classical theory is not correct.

However classical theory explains that photo current is proportional to intensity because the more energy carried by the wave front, the more energy will be absorbed by the surface electrons; and therefore more electrons will escape from the metal surface. Classical theory fails to explain experimental observations 3 and 4.

The quantum theory of photoelectric effect

Einstein postulated that light is emitted and absorbed in discrete amounts of packets called quanta or photons. The energy of each photon is hf, where f is frequency of light and h is Planck's constant.

When light on a metal surface, each photon of light interacts with one and only one electron in the surface of the metal giving it all its energy or none at all.

If the photon energy, hf, is greater than the work function of the metal, ω_0, electrons are dislodged from the attraction of the nucleus of the metal.

Work function is the minimum energy required to overcome the attraction of electrons by the nuclei of the metal surface. Different metals have different work functions. Work functions can be expressed in electron volt (eV).

Note: Electron volt is the kinetic energy gained by an electron when it is accelerated by a p.d of 1V.

Hence kinetic energy = $\frac{1}{2}mv^2$= eV = $1.6 \times 10^{-19} \times 1 = 1.6 \times 10^{-19}$J

1eV = 1.6×10^{-19}J

Electron emission occurs only if **$hf > \omega_0$**

The difference hf - ω_0 is available to the emitted electrons as kinetic energy . The maximum kinetic energy of photoelectrons is given by

$$\frac{1}{2}mv_{max}^2 = hf - \omega_0$$

$$\therefore hf = \frac{1}{2}mv_{max}^2 + \omega_0 \ldots\ldots(1)$$

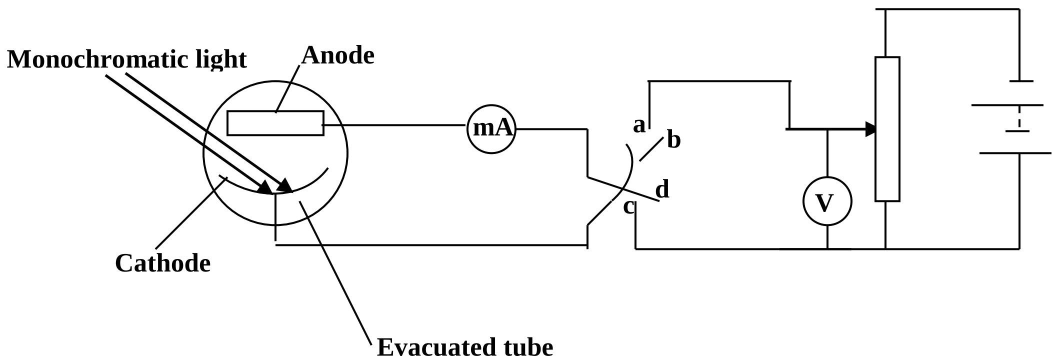
Equation (1) is called *Einstein's equation of photoelectric effect*.

$\omega_0 = hf_0$, where f_0 is the threshold frequency of the metal surface.

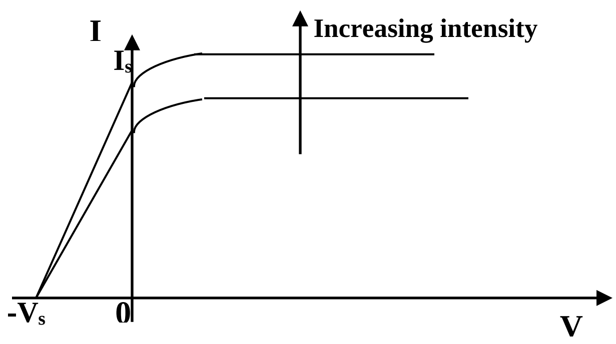
$$\frac{1}{2}mv_{max}^2 = h(f - f_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

where **λ** is the wavelength of the incident photon and **λ_0** is the threshold wavelength

Experimental investigation of photoelectric effect



With a connected to b and c to d, the photocurrent I is measured for increasing values of V. A graph of I against V is plotted and has the form shown below;



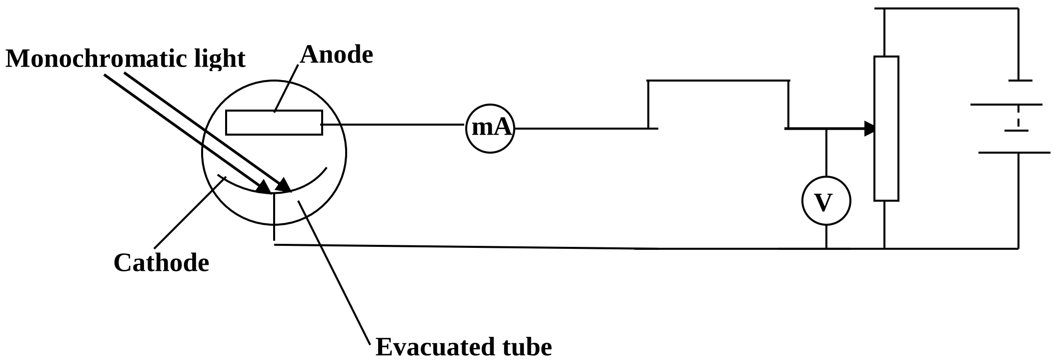
The curves saturate early meaning that a small voltages is sufficient to collect all the electrons emitted. The photocurrent is not zero when the voltage is zero. The photocurrent persists even when the anode is negative relative to the cathode. This is because electrons are emitted with sufficient kinetic energy which overcomes the opposing electric field and reach the anode.

If the anode is negative with respect to the cathode (i.e. when a is connected to d and b to c), and the applied p.d across the anode is varied, a stage is reached when electrons stop reaching the anode, hence no current flows. This negative potential at the anode at which the photo current is zero is called *Stopping potential* (Vs).

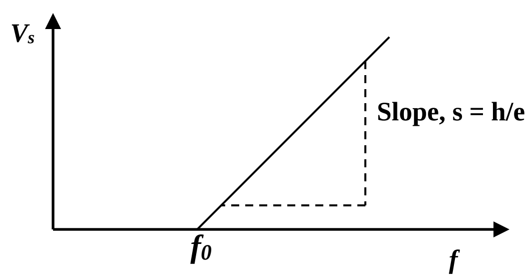
Hence at stopping potential, $\frac{1}{2}mv_{max}^2 = eV_s$ where V_s is the stopping potential.

Hence $eV_s = hf - \omega_0$

Experiment to determine Planck's constant



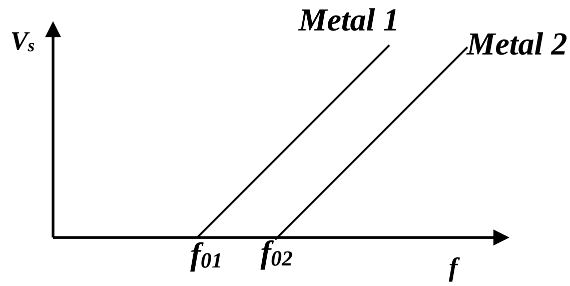
The anode is at a negative potential with respect to the cathode. The pd between the cathode and anode is varied until the photocurrent recorded by the ammeter is zero. The value of Vs of the voltmeter when this occurs is noted. The experiment is repeated for monochromatic light of different frequency, f but same intensity. A graph of Vs against f is plotted



The slope of the graph, s = h/e, where e is the electronic charge

Hence Planck's constant, h = se.

For different metal surfaces we obtain,



Examples

1. the minimum frequency of light that would cause photoelectric emission from Potassium surface is $5.37 x 10^{14}$Hz. When the surface is illuminated by another radiation, photoelectrons are emitted with a speed of $7.9 x 10^5 ms^{-1}$. calculate the:
   (i) work function of Potassium metal
   (ii) maximum kinetic energy of the photo electrons.
   (iii) Frequency of the second source

   (h = $6.625 x 10^{-34}$Js, mass of the electrons = $9.11 x 10^{-31}$kg)

   solution

(i) $\omega_0 = hf_0 = 6.625 \times 10^{-34} \times 5.37 \times 10^{14} = 3.558 \times 10^{-19} J$

(ii) Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times \left(7.9 \times 10^5\right)^2 = 2.84 \times 10^{-19} J$

(iii)
$$hf = \frac{1}{2}mv^2 + \omega_0 = 2.84 \times 10^{-19} + 3.56 \times 10^{-19} = 6.4 \times 10^{-19} J$$
$$f = \frac{6.4 \times 10^{-19}}{6.625 \times 10^{-34}} = 9.66 \times 10^{14} Hz$$

2. A 100mW beam of light of wavelength 400nm falls on a caesium surface of a photocell.

(i) how many photons strike caesium surface pr second.

(iii) If 80% of the photons emit photoelectrons, find the resulting current.

(iv) Calculate the kinetic energy of each photoelectron if the work function of caesium is 2.15eV.

Solution

(i) power = nx(energy of a photon) = $n \times hf = n \times \frac{hc}{\lambda}$, where n is number of electrons emitted per second

$$0.1 = n \times \left(\frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}} \right)$$

$$n = 2 \times 10^{17}$$

(ii) number of photons which produce electron emission= $\frac{80}{100} \times 2 \times 10^{17} = 1.61 \times 10^{17} = n'$, number of electrons emitted per second.

But $I = n'e = 1.61 \times 10^{17} \times 1.6 \times 10^{-19} = 0.026A$

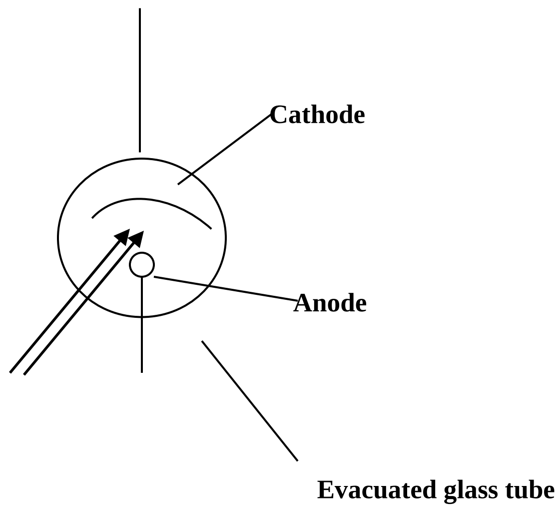
(iii) Kinetic energy $= hf - \omega_0 = 6.625 \times 10^{-34} \times \left(\frac{3 \times 10^{8}}{4 \times 10^{-7}} \right) - \left(2.15 \times 1.6 \times 10^{-19} \right) = 3.46 \times 10^{-20}\ J$

Exercise

1. The work function of a cleaned metal surface is 4.5eV . calculate
   (i) the minimum frequency of the radiation that will cause emission of electrons from the metal surface. (1.09×10^{15}Hz)
   (ii) The maximum energy of the electrons emitted when the surface is illuminated by radiation of frequency 1.2×10^{15}Hz. (7.5×10^{-20}J)

Application of photo electricity

A photoemmisive cell consists of two electrodes enclosed in a glass tube which may be evacuated or containing an inert gas at low pressure.



The cathode is curved metal plate having an emissive surface facing the anode. When electromagnetic radiation fall on the cathode, photoelectrons are emitted and are attracted to the anode if it is a suitable positive potential. A current of few microamperes flows and increases with the intensity of the incident radiation.

This photocell can be used to detect intruders. The intruder intercepts the infra red falling on the photocell. Hence current is cut off. The interruption therefore sets off the alarm.

Atomic Nucleus

Rutherford's model of the atom

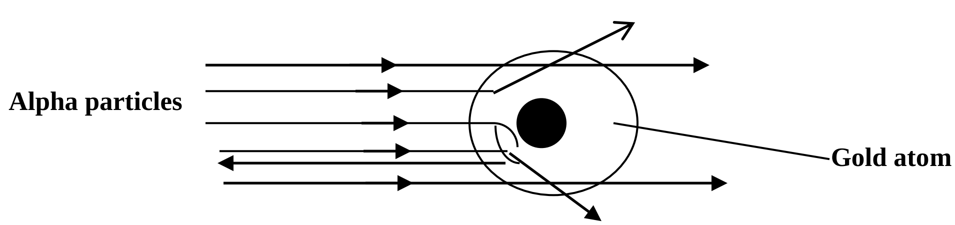
All the positive charge of the atom is concentrated in a small region called the nucleus of diameter less than 10^{-15}m. The negative charge surrounds the positive charge.

This was verified by Rutherfold and his team. The experiment involved the scattering of thin Gold foil.

Alpha particles emitted by a radioactive source were directed towards a thin gold foil. The scattered alpha particles were observed on a fluorescent screen on the focal plane of the microscope. Scintillations were observed on the screen whenever the alpha particles struck the Zinc Sulphide scintillation detector. The microscope was moved to different positions in order to detect the alpha particles.

Observations

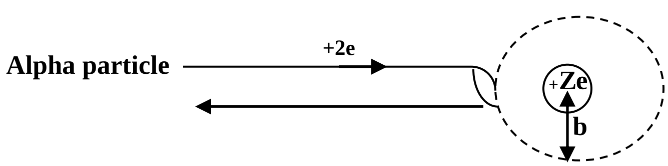
1. The majority of the alpha particles passed through undeflected.
2. A few of the alpha particles were scattered through small angles.
3. Very few alpha particles were deflected through angles greater than 90º.



Conclusion

1. The alpha particles being positively charged, their scattering must be due to the positive charge in the gold atom.
2. Since the majority of the alpha particles passed through undeflected, most of the space inside the atom is empty.
3. Large angle scattering occurred whenever an alpha particle was incident almost head on to the nucleus.
4. Since very few alpha particles were scattered through large angles, it follows that the probability of a head on collision with the nucleus is small and it follows that the nucleus occupies only small proportion of the available space inside an atom.

Closest distance of approach of alpha particles.



Where Z is the proton number or atomic number of the atom.

At closest distance of approach , all alpha particle's kinetic energy is converted into electrostatic potential energy of the alpha particle or nucleus system.

Hence w.d=Q_xV ; But V=$Q_\alpha/(4\Pi\epsilon_0 b)$; Q_α=2e ;Q_x=Ze ;

w.d=K.E=(1/2)mu^2 Therefore;

$$\frac{1}{2}mu^2 = \frac{2Ze^2}{4\pi\varepsilon_0 b}$$

$$\frac{1}{2}mu^2 = \frac{Ze^2}{2\pi\varepsilon_0 b}$$

$$b = \frac{Ze^2}{\pi\varepsilon_0 mu^2}$$

Example

A beam of alpha particles of energy 4.2MeV is incident normal to a gold foil. What is the closest distance of approach by the particles to the nucleus of the gold atom?

(Atomic number of gold = 79)

$$\frac{1}{2}mu^2 = \frac{Ze^2}{2\pi\varepsilon_0 b}$$

$$4.2\times 1.6\times 10^{-13} = \frac{79\times\left(1.6\times 10^{-19}\right)^2}{2\times\pi\times 8.85\times 10^{-12}\times b}$$

$$b = 5.412\times 10^{-14}\,m$$

Summary, the atom consists of the following main particle: (i) the protons which are positively charged, (ii) the neutrons which carry no charge and the electrons which are found in orbits around the nucleus. The neutrons and protons make up the nucleus of the atom.

Rutherford's model successes

i) It accounts for the observations of Alpha particle scattering by a metal foil.

ii) The radius of an atom can be estimated using the model.

FAILURES OF RUTHERFORDS MODEL

Rutherford's model failed to explain the existence of a stable atom with orbiting electron.

An orbiting electron is constantly changing its direction of motion and is therefore accelerating. This means that it would constantly emit electromagnetic radiation and its radius of the orbit would constantly reduce until it would spiral (spin) into the nucleus. If this happens for all electrons, the atom would cease to exist.

NB: Emission of e-m radiation by electrons implies the reduction in their energy which makes them fall into an orbit of a smaller radius and this would have continued until the electron enters the nucleus.

The Bohr model of the atom

Bohr postulated that:

(i) Electrons in atoms can exist only in certain discrete orbits and while in these orbits, they don't radiate (emit) energy.

(ii) Whenever an electron makes a transition from one orbit to another of lower energy, a quantum of electromagnetic radiation is given off.

The energy of the quantum of radiation emitted is given by $E = hf = E_i - E_f$, where E_i is energy of the electron in the initial orbit, E_f is the energy of the electron in the final orbit, h is Planck's constant and f is the frequency of emitted electron.

(iii) The angular momentum of an electron in its orbit in an atom is an integral multiple of $\frac{h}{2\pi}$

i.e. $mvr = \frac{nh}{2\pi}$, where n = 1, 2, 3 ....

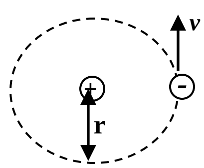
The orbit with the lowest energy is called the **ground state**. All physical systems are in physical equilibrium in the lowest energy state. Other high energy levels are called **excited state**.

The minimum energy required to remove an electron completely from the ground state (nucleus of an a tom or K-shell) is **called ionisation energy**

Bohr's atom simply means an atom with a small central positive nucleus with electrons moving around it only in certain allowed orbits and while in these orbits they don't emit radiations.

Bohr's theory of the hydrogen atom

Consider an electron in a hydrogen atom to be in a circular orbit of radius, r, about the nucleus.



For circular motion, a centripetal force on an electron is

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

Hence kinetic energy, $T = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$.................$(i)$

The electric potential energy of the electron, $V(r) = \frac{e}{4\pi\varepsilon_0 r} \times (-e) = \frac{-e^2}{4\pi\varepsilon_0 r}$...............$(ii)$

Total energy, E = T + V(r) = $\frac{e^2}{8\pi\varepsilon_0 r} + \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$..............$(iii)$

From Bohr's postulates, $mvr = \frac{nh}{2\pi}$

Hence $v^2 = \frac{n^2h^2}{4\pi^2m^2r^2}$................$(iv)$

Substitute equation (iv) in equation (i)

$$\frac{mn^2h^2}{8\pi^2m^2r^2} = \frac{e^2}{8\pi\varepsilon_0 r}$$

Hence $r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$.............$(v)$

Substitute equation (v) in equation (iii)

$$E = \frac{-e^2}{8\pi\varepsilon_0\left(\frac{n^2h^2\varepsilon_0}{\pi me^2}\right)} = \frac{-me^4}{8\varepsilon_0^2n^2h^2}$$

Hence the allowed electron energies can be obtained from the equation

$E_n = \frac{-me^4}{8\varepsilon_0^2n^2h^2}$, where n is the principal quantum number,; n – 1, 2, 3, ……..

Note: (i) The energy of the electron is always negative. This means that work has to be done to move the electron to infinity where it is considered to have zero energy. The electron is therefore bound to the nucleus.

(ii) Whenever an electron makes a transition from a higher energy level, n_i, to a lower energy level, n_f, the energy of the quantum of radiation emitted is

$$hf = E_i = E_f = \frac{-me^4}{8\varepsilon_0^2n_i^2h^2} - \frac{-me^4}{8\varepsilon_0^2n_f^2h^2} = \frac{me^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Energy of the ground state:- $E_0 = \frac{-me^4}{8\varepsilon_0^2h^2}$, since n = 1

But $m = 9.11\times10^{-31}, \varepsilon_0 = 8.85\times10^{-12}, h = 6.6\times10^{-34}$

$$E_0 = -2.18\times10^{-18}J$$
$$E_0 = -13.6eV$$

Hence $E_n = \frac{-13.6}{n^2}eV$

$$E = hf = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$f = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The wave number of the radiation emitted is $\sigma = \frac{f}{c} = \frac{me^4}{8\varepsilon_0^2 h^2 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

The term $\frac{me^4}{8\varepsilon_0^2 h^3 c} = R_H = Rydberg$ constant

$$\sigma = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Spectral lines of hydrogen atoms

Energy levels are grouped into shells. Electrons in one shell have nearly the same energy. The shells are denoted by letters K, L, M, N etc. where K corresponds to n = 1, L to n = 2, M to n = 3 and so on.

Transitions of electron from a high energy level to lower energy level cause electron to lose energy hence producing electromagnetic waves. Transitions from other shells to K- shell emit spectra of wavelength grouped into what is called Lymann series.

Lymann series lie in the Ultra violet region of the spectrometer.

$$\sigma = R_H\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$$

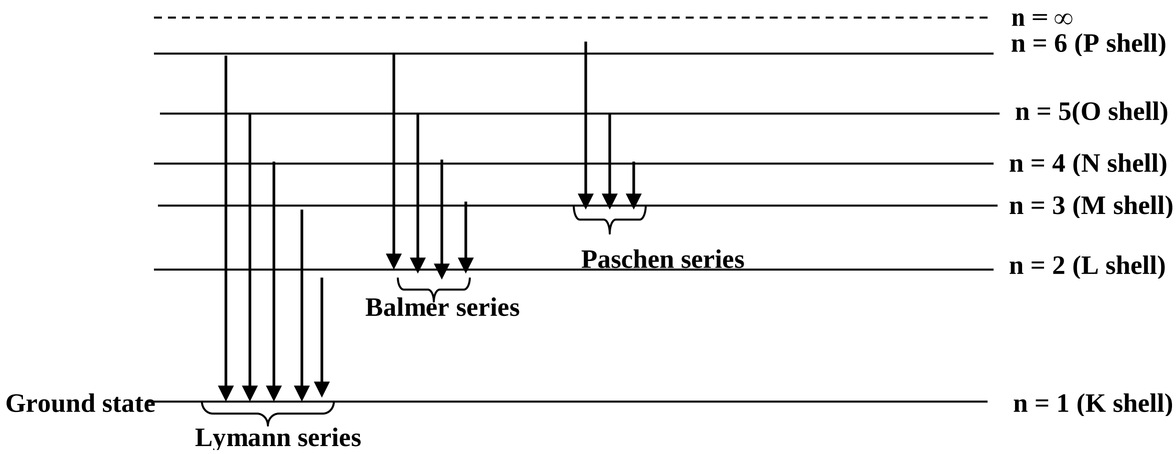
Where n_i = 2, 3, 4…….

Transitions from other high energy levels to the L- shell (n = 2), emits spectra of wavelengths referred to as Balmer series. Balmer series lie in the visible spectrum.

$\sigma = R_H\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$, $\quad$ n_i = 3, 4, 5…

Transition from other high energy levels to the M – shell (n = 3), emits spectra referred to as Paschen series which lie in the infra red region.

$\sigma = R_H\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$, n_i = 4, 5, 6……..



Failures of* Bohr's *model

- His theory fails to explain the relative intensities of spectral lines
- Bohr's theory fails to explain spectra of more complicated atoms (A toms having more than one electron)
- The method of selecting allowed orbits was arbitrary and lack theoretical background.

However, the following remain valid (successes):

(i) electrons exist outside the atomic nucleus

(ii) Existence of energy levels.

(iii) Emission and absorption of radiation occur in discrete amounts called quanta.

Line emission spectra

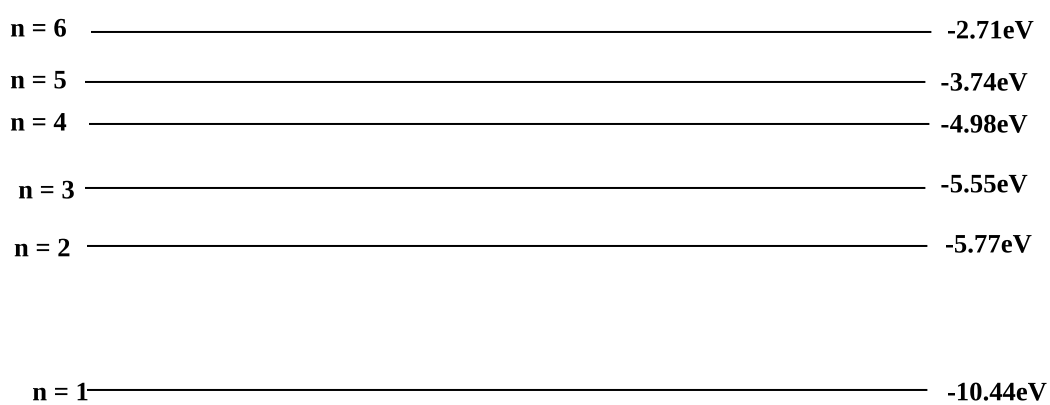
When atoms like H_2, neon etc. are excited due to some form of heat from a frame or electricity, electron transition may occur to higher energy levels. This makes the atom unstable since energy has increased. Electron transition may occur due to a vacancy left in the lower energy level and radiation of a definite wavelength or frequency is emitted. A line appears bright against a dark background. The lines are separated which give evidence that energy levels of the atoms are separate.

Line absorption spectra.

An atom's energy can change by only discrete amounts. If a photon of energy, hf, is just enough to excite the atom, such that an electron can jump to one of higher energy levels, the photon will be absorbed. The intensity of the incident radiation is reduced since it has lost a photon. A dark line on a white background is observed, whose wavelength is that of the absorbed photon.

Example

1. The figure below represents the lowest energy levels of mercury.



(i) Calculate the ionisation energy in eV and in joules

(ii) Calculate the energy and wavelength of the photon emitted when the mercury atom's energy changes from E_6 to E_2.

(iii) Determine which energy levels in the mercury atom are involved in the emission of a line whose wavelength is 546nm.

2. The figure below shows some of the energy levels of a neon atom. In what region of the electromagnetic spectrum does the radiation emit in the transition E_3 to E_2 lie?

3 A hydrogen atom is in excited state of energy -10.6ev. It absorbs a photon of wavelength 1.2 x 10^{-7}m and is exited to a higher energy level. When it falls back to its ground state, a photon of wavelength 0.9 x 10^{-7}m is emitted. Find the energy of the ground state.(4marks)

$E_{n'} - E_n = hc/1.2 \times 10-7$ where En = -10.6ev

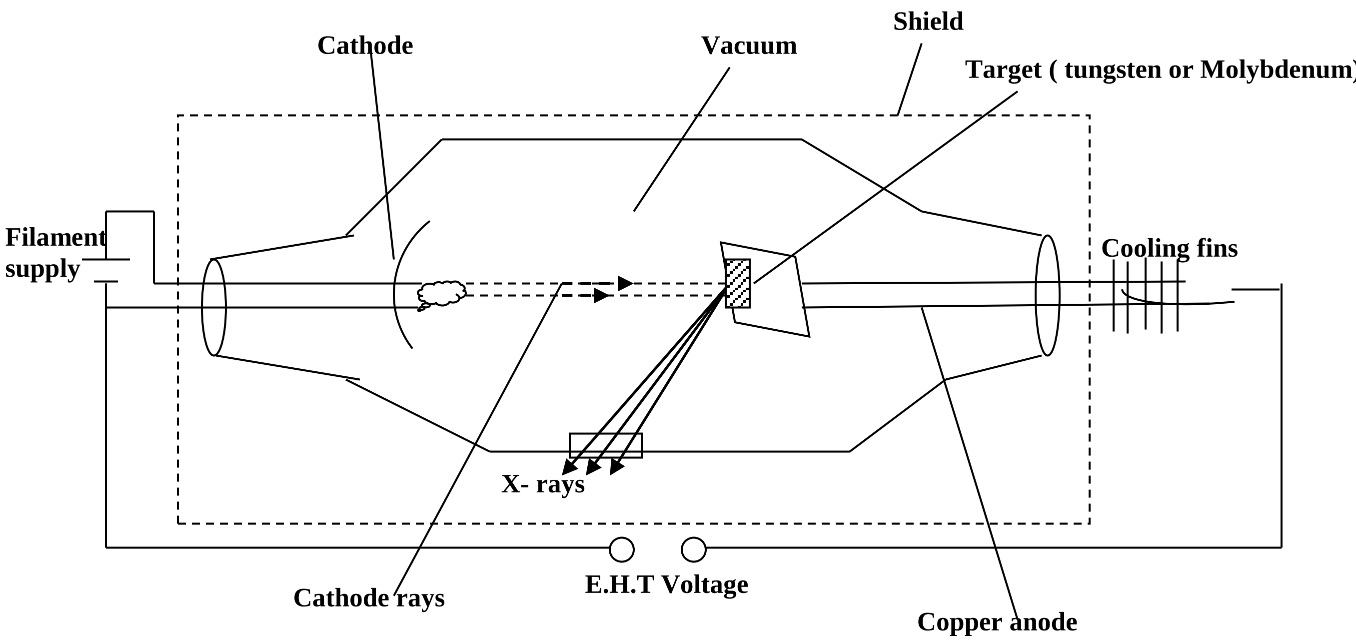
But; $E_{n'} - E_1 = hc/0.9x10-7$

$E_1 =$

X – Rays

X- rays are short wavelength electromagnetic waves which are produced when cathode rays are stopped by heavy metals.

Production of X – rays



Mode of operation

A low voltage is applied across the filament and heats the filament. Electrons are emitted by the filament by thermionic emission. The concave focussing cathode focuses the electrons from the filament onto the target. A very high alternating voltage is applied between the filament and the anode. During the half cycles when the anode is at a positive potential relative to the cathode, electrons are accelerated across the tube. No electrons flow to the anode when the anode is at a negative potential relative to the cathode.

When the cathode rays (electrons) strike the target, 99% of the kinetic energy of electrons is dissipated into heat while 1% is turned into X-rays.

The heat generated at the target is cooled by means of the copper cooling fins mounted on the copper anode. Heat is conducted from the target away from the tube by conduction and radiation.

The electron current, I in an X-ray tube in operation is given by I = ne, where n is the number of electrons per second and e is the electronic charge.

Intensity of X-rays (Quantity)

The intensity of X- rays in an X – ray tube is proportional to the number of electrons colliding with the target. The number of electrons produced at the cathode depend on the filament supply. The greater the heating current, the greater the number of electrons produced and hence more x- rays are produced. Therefore the intensity of X- rays is controlled by the filament current.

Penetration of X – rays (quality)

Penetration power of X-rays depends on the kinetic energy of the electrons striking the target. The higher the accelerating voltage, the faster the electrons produced. Faster electrons posses higher kinetic energy and shorter wavelength x-rays of greater penetration power are produced. Hence penetrating power of X-rays is determined by the accelerating Voltage across the tube.

Hard and soft X- rays

Hard x-rays have a high penetrating power. This because they have very short wavelengths. They are produced when a high p.d is applied across the tube.

Soft X-rays are produced by electrons moving at relatively lower velocities than those produced by hard x –rays. They have less energy, longer wavelengths, hence less penetration power compared to hard x-rays.
Hard x-rays can penetrate flesh but are absorbed by bones. Soft x-rays are used to show malignant growths since they only penetrate soft flesh. They are absorbed by such growths.

Properties of X –rays

- They travel in a straight line at a speed of light
- They are not deflected by both magnetic and electric fields. This indicate that they carry no charge.
- They penetrate all matter to some extent. Penetration is least in materials with high density and atomic number e.g. lead.
- They ionise gases through which they pass.
- They affect photographic plates just like light does.
- They cause fluorescence in some materials.
- They cause photoelectric effect when they are illuminated on certain metal surfaces.
- They are diffracted by crystals leading to an interference pattern.

Examples

In an x-ray tube 99% of the electrical power supplied to the tube is dissipated as heat. If the accelerating voltage is 75kV and power of 742.5W is dissipated as heat, find the number of electrons arriving at the target per second.

$\frac{99}{100} \times power = 742.5W$

Hence power supplied = $\frac{742.5 \times 100}{99} = 750W$

But power = VI

Hence 75000I = 750

I = 0.01A

But I = ne

1.01 = 1.6×10^{-19}n

Therefore n = 6.25×10^{16} per second

Exercise

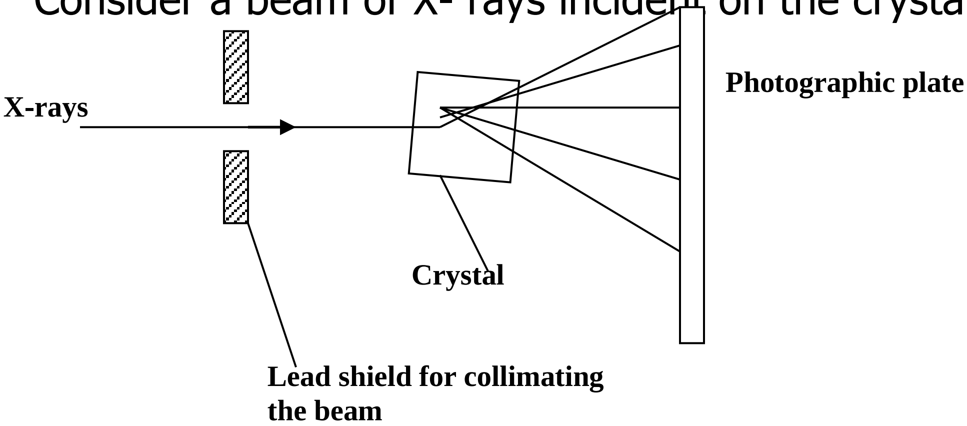
1. In an x-ray tube operated at 5×10^5V, the target is made of material of specific heat capacity $2.5 \times 10^2 Jkg^{-1}K^{-1}$ and has a mass of 0.25kg. 1% of the electrical power is converted into x-rays and the rest is dissipated as heat in the target. If the temperature of the target rises at $8Ks^{-1}$, find the number of electrons which strike the target every second.

2. The current in a water- cooled x-ray tube operating at 60KV is 30mA. 99% of the energy supplied to the tube is converted into heat at the target and is removed by water flowing at a rate of $0.060kgs^{-1}$. Calculate: (i) the rate at which energy is being supplied to the tube. ($1800Js^{-1}$)

(ii) the increase in temperature of the cooling water, assume specific heat capacity of water = $4200Jkg^{-1}K^{-1}$. (7.1°C)

Diffraction of X-rays by crystals

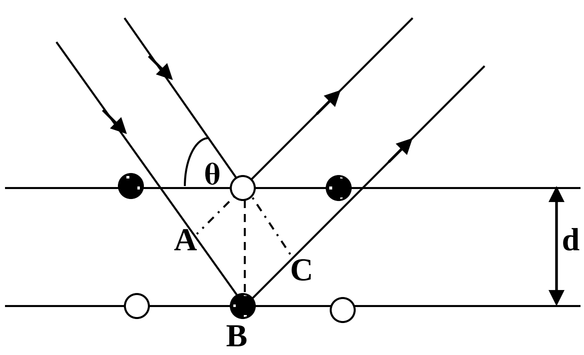
Consider a beam of X- rays incident on the crystal.



After long exposure to x-rays, the photographic plate is developed and printed. A regular pattern of dark spots called Laue spots are observed around the central dark image. The pattern is due to scattering of x-rays by interaction with electrons in the atom of the crystal. The regularity of the spots implies that atoms in the crystal are arranged in a regular pattern.

Consider a beam of monochromatic x-rays incident on a crystal such as KCl at a glancing angle θ.

Let d be the interatomic or interplanar spacing and λ the wave length of the x-rays. A small fraction of the incident x-rays is scattered by each atom. The scattered x-rays interfere constructively in those directions for which the angle of incidence is equal to the angle of reflection.



The path difference between x-rays scattered by atoms in two consecutive planes = AB + BC.

But AB = BC = dsinθ

For constructive interference path difference AB + BC = nλ,

where n = 1, 2, 3, ….. is called the order of diffraction and θ is the glancing angle.

Hence

2dsinθ = nλ ……. For n = 1, 2, 3... .This is Bragg's law

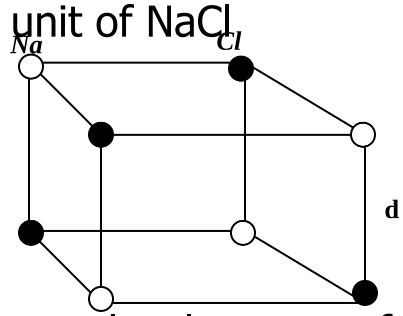
Example

X-rays of wavelength 10^{-10}m are diffracted from a set of planes of rubidium Chloride. The first diffraction maxima occurs at 8.8°. calculate the interplanar spacing.

$$2d\sin\theta = n\lambda,$$
$$n = 1, \lambda = 10^{-10}$$
$$2d\sin 8.8 = 10^{-10}$$
$$d = 3.27\times 10^{-10}\,m$$

Atomic spacing in crystals

Consider a unit of NaCl



Let M be the molecular mass of NaCl, ρ density and N_a Avogadro's number.

Mass per molecule $= \frac{M}{N_a}$

Volume associated with one molecule $= \frac{M}{N_a \rho}$

One molecule of NaCl has two atoms, hence volume associated with one atom = $\frac{M}{2N_a\rho}$

But volume associated with one atom = d^3.

Hence $d^3 = \frac{M}{2N_a\rho}$

Example

A beam of x-rays of wavelength $1.0\text{x}10^{-10}$m is incident on a set of cubic planes of NaCl crystal (Molecular mass = 58.8). First order diffraction is obtained for a glancing angle of 10.2°. find

(i) the spacing between consecutive planes

(ii) the density of NaCl.

Solution

(i)
$$2d\sin\theta = n\lambda$$
$$n = 1, \lambda = 10^{-10}, \theta = 10.2$$
$$d = \frac{10^{-10}}{2\times\sin 10.2} = 2.82\times10^{-10}m$$

(ii)
$$d^3 = \frac{M}{2N_a\rho}$$
$$\rho = \frac{58.8\times10^{-3}}{2\times\left(2.82\times10^{-10}\right)^3\times6.02\times10^{23}} = 2.16\times10^3 kgm^{-3}$$

Question

A monochromatic beam of x-rays of wavelength $2\text{x}10^{-10}$m is incident on a set of cubic planes in a KCl crystal. First order diffraction maxima are observed at a glancing angle of 18.5°. Find the density of KCl if its molecular weight is 74.55g. ($1.97\text{x}103\text{kgm}^{-3}$)

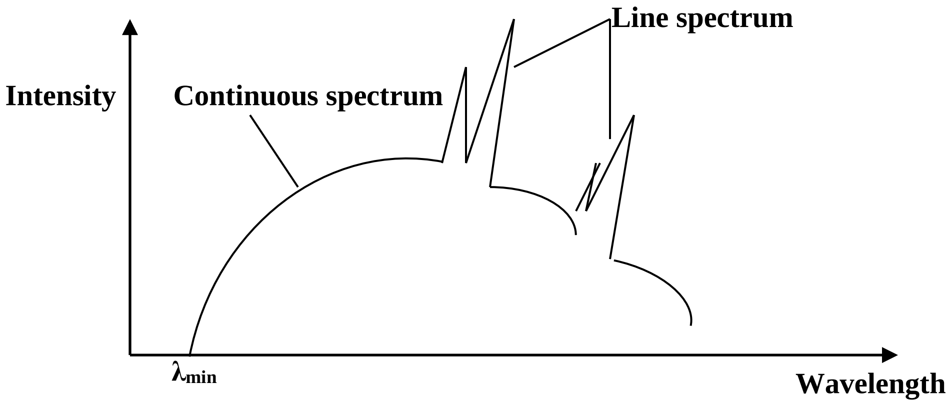
X-ray spectra in an x-ray tube

There are two spectra; continuous and line/prominent/ characteristic spectra.

The intensity of x-rays plotted as a function of wavelength has the features shown below. The line spectrum is superimposed on the continuous spectrum.

Continuous spectrum

This arises from multiple collisions of electrons with target atoms. Different amounts of energy are lost during these collisions. The x-rays given off when the electrons are decelerated will have wavelengths varying from a certain minimum value λ_{min} to infinity.



When an electron loses all its energy in a single collision with an atom of the target, a most energetic x-ray photon is given off. The kinetic energy of the electrons equal to eV, where V is the accelerating voltage between a filament and the anode. It is converted into electromagnetic radiation of energy, $hf_{\max} = \frac{hc}{\lambda_{\min}} = eV$

Hence $\lambda_{\min} = \frac{hc}{eV}$ ....(1)

Equation (1) is called the Hunt- Duane equation . λ_{min} represents the minimum wavelength of the X-ray produced for a given accelerating voltage V. It is also called-cut off wavelength.

Line spectrum

Uses of X-rays

1. Structural analysis, stresses, fractures in solids, castings and welded joints can be analysed by examining X-ray photograph.
2. Crystallography; Orientation and identification of minerals by analysis of diffraction patterns using Bragg's law.
3. Medical uses;

   (i) Analytical uses. These include location of fractures, cancer and tumour/defective tissue absorbs x-rays differently from normal tissue.

   (ii) Therapeutics use for destroying cancerous cells and tumours.

5. detection of fire arms at international airports.

Nuclear physics

The nuclei of atoms contain protons and neutrons. The collection of protons and neutrons together is called the nucleon.

A species of atoms with a specified number of protons and neutrons is called a nuclide. There are forces which bind the nucleons together. In some nuclides, the forces make the nucleons stay together permanently; however in some, the energy forces binding the nucleus affect some to the nucleons, this happens when the ration of neutrons to protons is big. When ration is big, the nucleus release excess energy to become stable.

-The number of protons in the nucleus is called the **atomic number**

-The number of protons and neutrons is the **mass number (Atomic mass).**

An atom X, with atomic number Z and mass number A can be symbolised by $^{A}_{Z}X$

A = Z+N, where N = number of neutrons

RADIOACTIVE DECAY

This is the spontaneous disintegration of unstable nuclei emitting alpha, α, beta, β particles and gamma, γ radiation

Alpha particles

An α-particle is a Helium atom that has two protons and two neutrons. When a nuclei decays by release of an α particles, it loses two protons and two neutrons i.e. mass number decreases by 4 and atomic number by 2. Alpha particle symbol is $^{4}_{2}He$

$$^{A}_{z}Z \rightarrow ^{206}_{82}He + ^{A-4}_{z-2}Y$$

eg

$$^{210}_{84}Po \rightarrow ^{206}_{82}Pb + ^{4}_{2}He$$

Properties of ɑ particles

-They cause fluorescent in some materials

-They blacken photographic plates

-They readily ionise gases

-They are easily absorbed by matter.

-The penetration of matter by ɑ particles is unique in that the ɑ particles can not be detected beyond their range.

-They are deflected by electric and magnetic fields to a less extent than β particles. This means that they are heavier than β particles. In both magnetic and electric fields they are deflected in a direction opposite to that of the β particles. This indicates that they are positively charged.

-They are emitted with speeds of the order $10^7 ms^{-1}$

-They are helium nuclei with mass 4U and charge +2e

Beta particles

These are electrons, the mass of the electron is much smaller than that of the proton

When an element decays by emitting a β particle it loses an electron. Hence the mass number remains the same but the atomic number increase by one.

A neutrons is thought to consist of a proton and an electron. When a nucleus undergo a disintegration, a neutron breaks down into an electron

(β particle) which is emitted and a proton which increases the atomic number.

Properties of β particles

They have a much smaller fluorescent effect than $\propto$particles

They blacken photographic plates

They ionise gases readily than $\propto$ particles

They penetrate matter more easily than ɑ particles but are absorbed completely by about 1mm of Aluminium, or a few metres path of air.

They don't have a definite range like ɑ particles owing to successive deflection caused by collision with the atom of the absorber.

-They are deflected by electric and magnetic fields much more than $\propto$ particles because they are lighter.

They are fast moving electrons. They move faster than alpha particles.

Gamma rays

They are electromagnetic radiation with very short wavelengths. These are found to occupy a band before the X-ray which are thought to have the shortest ware length known.

The main difference between δ- rays and X-rays is that δ -rays originate from energy changes in the nucleus in the atom while X- rays originate from energy changes associated with electron structure of the atom.

Emission of δ rays has no effect on the mass of the nucleus. Emission of δ- rays is usually accompanied by ɑ or β emission e.g.

$$^{234}_{90}Th \rightarrow ^{234}_{91}Pa + ^{0}_{-1}\ell + \delta$$

$$^{60}_{27}Co \rightarrow ^{60}_{28}Ni + ^{0}_{-1}\ell + \delta$$

Properties

-Affect photographic plates

-They are not deflected by magnetic and electric fields. This implies that they carry no charge

-They travel in a vacuum with the speed of light

-They have the highest penetrating power because of their light mass and due to this they can be stopped or absorbed by a lead metal or shield which has the highest density.

-They cause photoelectric effect i.e. they eject electrons when they fall on certain metals.

-They can also cause ionisation of a gas by knocking off electrons from the neutral atoms but this is by small amounts.

-They have the highest possible range in air.

Note:

(i) Range of radiation is the maximum distance covered by a radiation in air before it is totally absorbed.

(ii) Ionisation is the process of changing the neutral atoms of a gas into positive and negative ions.

The Decay law

It states that the rate of disintegration of a given sample at any time is directly proportional to the number of nuclide N, present at that time, t.

Mathematically

$$\frac{dN}{dt} \propto (-N)$$

The negative sign indicates that N decreases as t increases

$$\frac{dN}{dt} = -\lambda N \quad \text{Where } \lambda \text{ is the decay constant}$$

Decay constant, λ, is defined as the fraction of the radioactive nuclei which decays per second.

$$\int \frac{dN}{N} = \int -\lambda dt$$
$$InN = -\lambda t + c$$

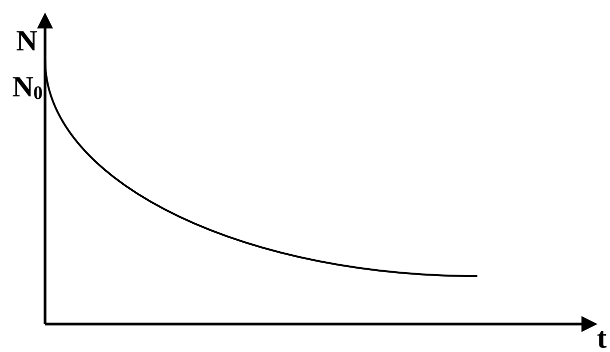
When t = 0, N = N_0, which is the original number of nuclei.

Hence $lnN_0 = c$

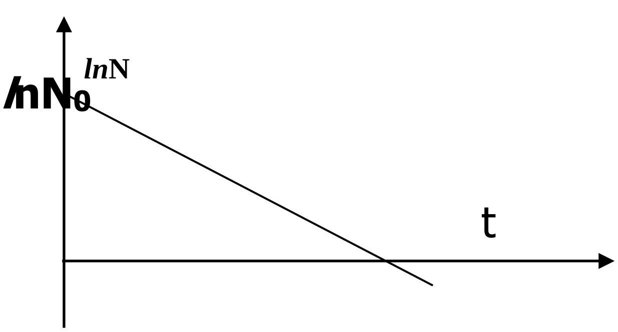
Hence $lnN = -\lambda t + lnN_0$

$$\ln\left(\frac{N}{No}\right) = -\lambda t$$
$$or$$
$$N = N_0 e^{-\lambda t}$$

A graph of N against t is called the decay curve



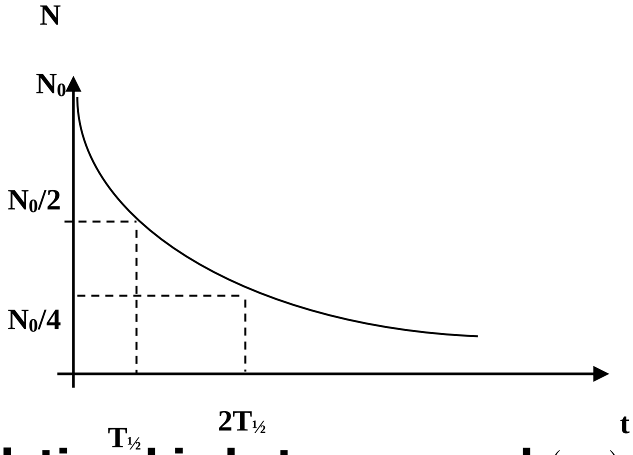
A graph of *ln*N against *t* is a straight line with a negative slope.



Half life $(T_{1/2})$

The half life of a radioactive source is the time taken for half the number of radioactive nuclei present in the source to disintegrate.

Consider the decay curve of a radioactive source



Relationship between λ and $(T_{1/2})$

When t = **T½ ,** N = $\frac{N_0}{2}$

From

$$N = Noe^{-\lambda t}$$

$$\frac{No}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$In\,1/2 = Ine^{-\lambda T_{1/2}}$$

$$In\,1/2 = -\lambda T_{1/2}$$

$$-0.693 = -\lambda T_{1/2}$$

$$\boxed{\lambda = \frac{0.693}{T_{1/2}} = \frac{\ln 2}{T_{1/2}}}$$

Activity of a radioactive source

This is number of disintegration of a radioactive source per second.

$$\text{Activity } A = \frac{dN}{dt} = -\lambda N$$

The SI unit of activity is Becquerel (Bq)

IBq = 1disintegration per second

A large unit of activity is curie (Ci)

$3.70 \times 10^{10} Bq = 1Ci$

$\text{Activity, } A = \frac{dN}{dt} = -\lambda N$

$N = Noe^{-\lambda t}$

$A = -\lambda Noe^{-\lambda t}$

but $A_0 = -\lambda N_0 = \text{initial activity}$

when $t = 0$

hence

$$A = A_0 e^{-\lambda t}$$

Hence Half-life can also be defined as the time taken for the activity of the source to decrease to half the original value.

Example

1. The half life of a radio isotope is 5.27 years, calculate
   i. Its decay constant
   ii. The number of years it will take 75% of a given mass of isotope to decay

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27 \times 365 \times 24 \times 3600} = 4 \times 10^{-9} s^{-1}$$

(ii)

$$N = Noe^{-\lambda t}$$

$$N = 0.25No$$

$$0.25No = Noe^{-\lambda t}$$

$$In0.25 = -\lambda t = 4 \times 10^{-9} t$$

$$t = 3.31 \times 10^{8} s$$

$$t = 10.5 years$$

2. The radio isotope ^{60}Co decays by emission of a β particle and a δ ray. Its half life is 5.3 years. Find the activity of the source containing 0.10 gm of ^{60}Co

$$A = \lambda N$$

$$but, \lambda = \frac{In \frac{1}{2}}{5.3 \times 365 \times 24 \times 3600} = 4.15 \times 10^{-9}$$

$$0.10 \text{ gm contain } \frac{\text{NA}}{60} \times 0.10 = \frac{6.02 \times 10 \times 0.10}{60} \textit{atoms}$$

$$N = 1.003 \times 10^{21} \textit{atoms}$$

$$A = \lambda N$$

$$A = 115 \times 10^{-9} \times 1.003 \times 10^{21} = 4.16 \times 10^{12} \text{ disntegrat ion s}^{-1}$$

Exercise:

A silver isotope $^{108}_{47}Ag$ has a half life of 2.4mins. Initially, a sample contain 2.0×10^6 nuclei of silver. Find the number of radioactive nuclei left after 1.2 minutes. (ans:1.412×10^6)

2.Aradioactive source contains 1.0µg of plutonium of mass number 239. If the source emits 2300 ɑ-particles per second, calculate the half-life of plutonium. (assume the decay law $N = N_0 e^{-\lambda t}$) Ans; 24073.99years

3. The mass of 1.0µg of $^{25}_{11}Na$ decays by emission of beta particles. If its half life is 60s, find the;

(a) initial activity of the sample

(b) number of atoms present after 10minutes.

A$_o$ =2.782E14; N = 2.285E13

4. $^{44}_{19}K$ has a half life of 20minutes. Find;

(a) the activity of the sample whose mass is 10$_m$g. Ans;7.903E16

(b) the activity of the sample after 1hour

Ans;A = 9.6288E17

CARBON DATING

The unstable isotope ^{14}C produced during nuclear reactions in the atmosphere as a result of cosmic ray bombardment give a small portion of ^{14}C in CO_2 in the atmosphere.

Plants take in CO_2 for photosynthesis. When a plant dies it stops taking in CO_2 and its ^{14}Cdecays to ^{14}N by β particle emission.

By measuring the activity A of ^{14}C in the remains, and the activity of the living or fresh sample A_o ,the time when the plant died can be estimated from;

Example

The activity of a sample of dead wood is 10 counts per minute, while for a living plant is 19 counts per minute. If the half life of ^{14}C is 5500 years, find the age of the wood sample.

$$A = A_0 e^{-\lambda t}$$
$$10 = 19 e^{-\lambda t}$$
$$-\lambda t = \ln\left(\frac{10}{19}\right)$$

but

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5500} yr^{-1}$$

Hence

$$-\frac{\ln 2}{5500} t = \ln\left(\frac{10}{19}\right)$$
$$t = 5093 \, years$$

Exercise

Wood from a buried ship has a specific activity of $1.2 \times 10^2 Bqkg^{-1}$ due to ^{14}C, whereas comparable living wood has an activity of $2 \times 10^2 Bqkg^{-1}$. What is the age of the ship? (half life of ^{14}C = $5,7 \times 10^3$years).

Radio isotopes

Radioisotopes are nuclides which are unstable and undergo radioactive decay emitting $\propto$ or β particles or γ- rays during return to a stable form.

^{238}U, ^{226}Ra and ^{230}Th are examples of natural radioactive.

A greater number of radio isotopes are produced artificially by bombarding stable nucleus with high energetic particles such as protons, α-particles, deuterons and neutrons.

Artificial radioisotopes behave the same way as the natural radioactive materials in that each will emit its characteristic particle or radiation and each has a characteristic half-life.

Examples

1. By bombarding ^{27}Al with α particles, one gets ^{30}P which decays by emission of a positron $\left({}^{0}_{1}e\right)$

   $${}^{27}_{13}Al + {}^{4}_{2}He \rightarrow {}^{30}_{15}P + {}^{1}_{0}n$$
   $${}^{30}_{15}P \rightarrow {}^{30}_{14}Si + {}^{0}_{1}e$$

2. Bombarding of boron ${}^{11}_{5}B$ with $\propto$ particles to get ${}^{14}_{6}C$ which decays by emission of β particles.

   $${}^{11}_{5}B + {}^{4}_{2}He \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$$

   then ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$ with half life 5730 years.

   -Neutrons are ideal for bombardment of stable nuclei to produce radioisotope because they carry no charge and are therefore not deflected by either atomic electrons or nuclear charge.

   They will penetrate the nucleus even when their energies are comparatively low.

Some uses of radioisotope

1. ***Biological uses***

i. Radiotherapy

-Radio cobalt $^{60}_{27}Co$ decays with emission of β particles together with very high energy γ-rays. The γ -rays have greater energy than is available with standard X- rays machines when properly shielded, the γ -rays are employed in the treatment of cancer.

-The iodine isotope ^{131}I (half life 8 days) decays by γ -ray emission. This is injected into the blood stream of a patient having cancer of the thyroid and the γ -rays given off are concentrated right where they are needed. The speed with which the iodine isotope becomes concentrated in the thyroid provides a measure of the thyroid function.

ii. Tracers

Small qualities of low activity radioisotope are administered by injection into patients and their passage through the body and absorption by diseased tissue studied.

The radioisotope ^{59} Fe is administered into a patient's blood stream. Measurement of the activity of blood sample from the patient and comparing it with initial activity of the radioisotope. The volume of blood in the patient can be determined. (In blood assessment, total volume in the sample, $V_T = \frac{VA_e}{A_v}$ where V_T –total volume of blood the patient has, V-volume of radioactive blood taken from the patient, A_e-expected activity after a time t, A_v-activity of radioactive blood. **Example;**

- A small volume of solution which contains radioactive isotope of Na had an activity of 12000dis/min when it was first injected into the blood stream of a patient. After 30hours, the activity of 1.0cm^3 of the blood was found to be o.5dis/min. If the half life of the isotope is 15hours,

(a) Estimate the volume of blood the patient has.

(b) Does the patient need any blood transfusion? (Explain your answer)

Solution;

At t = 0; A_o =1200dis/min

At t=half-life= 15hrs; A=6000dis/min

At t = 30hrs; A_e =3000dis/min

Therefore the expected activity after 30hrs is 3000dis/min

If 0.5dis/min (A_v) contains 1.0cm^3

Then3000dis/min contains $V = \frac{1.0x3000}{0.5}$ =6000cm^3=6litres

-There's no need for a blood transfusion since for a normal person has 5-6litres of blood.

In agriculture, tracers have` been used to study how fertilizers, hormones, weed killers and pesticides perform their functions. E.g. the radioisotope $^{30}_{15}P$ has been used to provide information about the best type of phosphate fertilizer to supply to particular crops and soil.

iii. Mutants

Radioisotopes have been used to induce plant mutations. This has led to improved seed varieties of crops like wheat, peas, and beans with high yields and high resistance to crop disease.

iv. Sterilization

Medical instruments and equipments are sterilized by exposure to γ – rays. Gamma ray as are also being used to sterilize and preserve some food products. The method is safe as no radioactivity is induced in the material irradiated by γ- rays.

Radiation has also been used to eliminate agricultural pests by sterilizing them and therefore breaking the reproduction chains.

v. Carbon dating

By measuring the activity of carbon-14 in the dead sample and comparing it with the activity of carbon-14 in a living sample, we can determine how long ago the organism died.

2. Industrial uses

(i) Tracers

a) For investigation of flow of liquids in chemical plants or in underground water and sewerage pipelines. In the latter cases, a little radioactive solution is added to the liquid being pumped. Temporary high activity around a leak is detected from the ground above. The rate of flow of liquids can also be measured.

b) For study of wear in machinery such as of piston rings in motor engine. Before the piston is put in place, it is irradiated with neutrons to form the radioisotope
^{59}Fe. As the piston rings wear out, it accumulates in the oil and by measuring the activity of ^{59}Fe in the oil and comparing it with the initial activity, the rate of wear of the piston is determined.(example; Steel piston rings contains 15g of a radioactive iron $^{54}_{26}Fe$ of activity 3.7×10^5dis/s. After 40 days of continuous use, the tank case oil was found to have a total activity of 1.2×10^3dis/s.

(i) Find the half-life of $^{54}_{26}Fe$ (3.122x10^{17}sec)

(ii) The average mass of iron worn off the ring per day assuming that all the metal removed accumulates in the oil.(1.25x10^{-3} g/day)

c) Automatic control of thickness paper, plastic or metal sheeting as it goes through the production plant. The thickness is controlled by measuring the transmission of radiation through the sheet.

3. Diagnostic uses

Cobalt ^{60} and other γ- rays' emitters are used as alternatives to X rays set ups which are more elaborate to produce radiographs for examination of welded beams and metal castings.

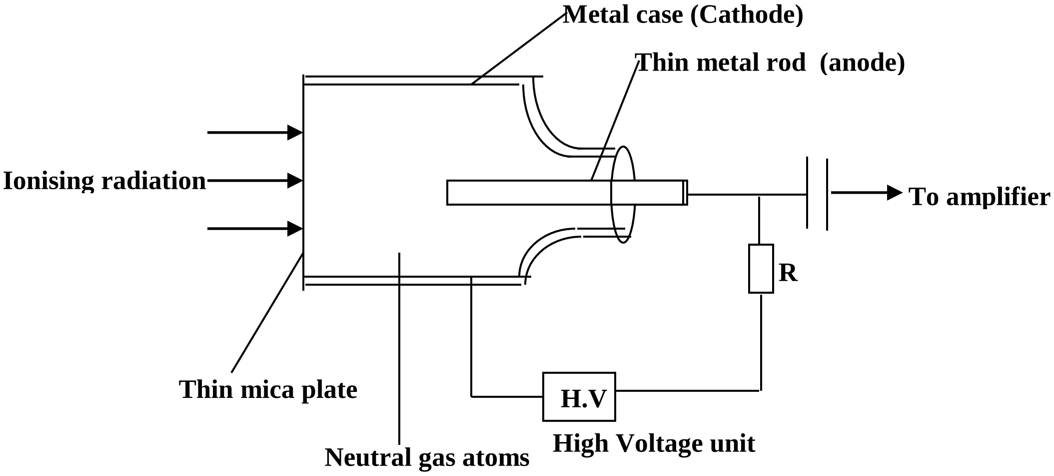
Detections of Nuclear radiation

The requirement for the detection of any nuclear radiation is that it must dissipate energy in the detector.

The causes of dissipation of energy by nuclear radiation are

i. Ionisation of atoms in the detector e.g in ionisation chamber and G.M. Tube
ii. Excitation of atoms without removal of orbital electrons e.g. in scintillation counter.

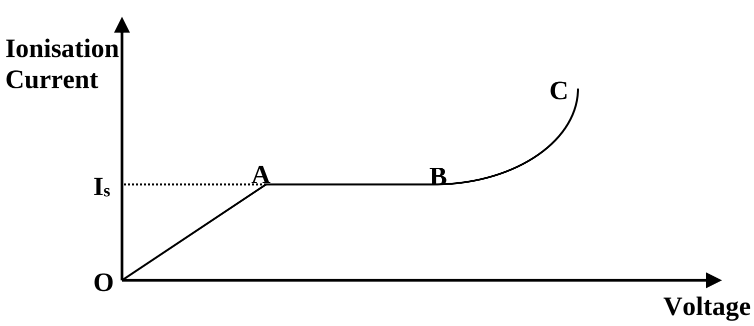
Ionisation chamber



Mode of action

When ionising radiation enters the chamber through the mica window, it ionises the neutral gas atoms. Ion pairs are produced as a result of collision. The positive ions produced drift to the cathode and the negative ions to the anode resulting in an ionisation current which is amplified and measured with the micrometer. A high voltage is set to a value that a constant current I_s flows. In this setting, the energy (intensity) of the incoming radiation is proportional to I_s.

A graph of ionisation current against voltage V has the following features.



Features of the graph

Region OA: The applied voltage is low. The positive ions and electrons produced by collisions of in coming radiation with the neutral gas atoms have a high chance of recombining since their velocities are low. The current produced is proportional to the voltage.

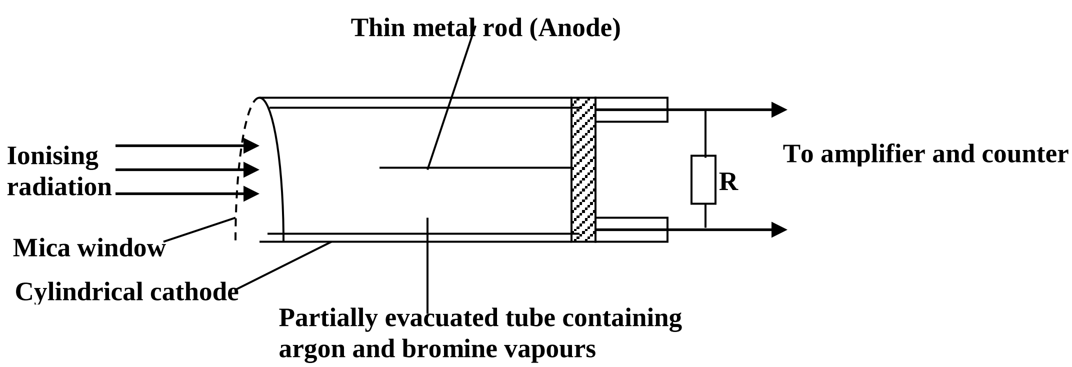
Region AB (*Saturation region*): All ions pairs produced per second travel and reach the respective electrodes. This results in constant current or saturation current $I_s = ne$.

$n = E_\alpha / E_I$ where E_α –Energy of 1-alpha particle ,E_I -Energy required to produce an iron pair, n-number of electrons produced.

In this region, the energy lost by the incoming particles is proportional to I_s. This is the region in which the ionisation chamber is normally operated.

Region BC (Gas amplification): because of the high voltage, the electrons produced by ionisation of the neutral gas atoms, acquire sufficient energy to cause ionisation themselves (secondary ionisation). This results in rapid multiplication of ions in the chamber, and hence the rise in ionisation current.

The Geiger Muller tube or counter

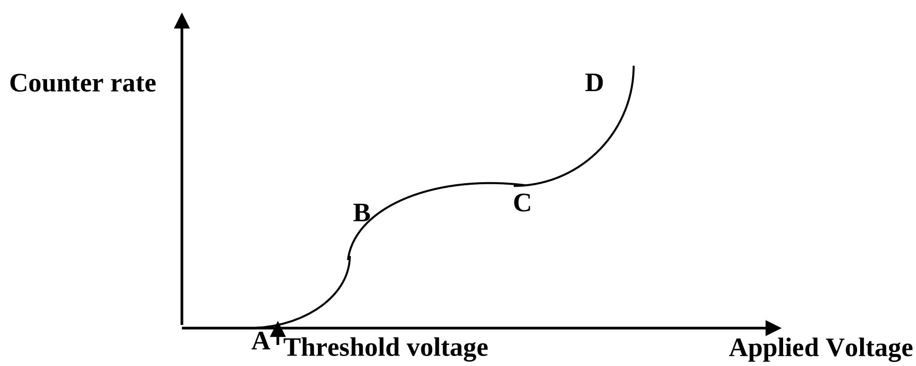


When the radiation enters the tube, it causes ionisation of the gas atoms. The electrons produced are accelerated to such a high energy that they also cause more ion pairs by repeated collisions. When the electrons reach the anode, the pulse is produced which is amplified and detected by a rate meter? The positive ions in the chamber are accelerated towards the cathode and if these ions reach the cathode, they will cause secondary discharge, which will give a false impression of an arrival in the chamber of another ionising particle. This is prevented by using a quenching agent like bromine.

During the quenching process, molecules of a quenching agent absorb the energy of the positive ions on collision.

In a G.M tube the time taken by the positive ions to travel to the cathode is known as the dead time. The number of pulses per unit time (counter rate) corresponds to the activity of the source of radiation

A graph of counter rate against applied voltage



Below a certain value of p.d known as *threshold voltage*, no counts are recorded at all since the number of ions produced per second is not enough to produce sufficient current which can result in a pd of sufficient magnitude to be detected.

Between A and B, magnitude of pulse developed in the tube depends on the initial ionisation of the tube and energy of the incident particles.

Between B and C, the plateau region, the counter rate is almost constant. This is the region when the tube is said to be normally operated

Beyond C, the counter rate increases rapidly with voltage due to incomplete quenching one incident particle may start a whole chain of pulses.

Background Radiation

-In using the G.M tube the background count rate is first recorded. The background count rate is due to the cosmic rays in the atmosphere, stray x-rays from hospitals and gamma rays from industries. The activity in the presence of the source is then determined and the difference is the actual activity due to the source at that point.

$A_s = A_r - A_b$

-If a G.M tube is placed at a distance R from a radioactive sample and the count rate A_s due to the sample is obtained, then

$A = \frac{4\pi R^2 A_s}{S}$ Where S - is the surface area of the mica window (A- is the activity free from errors due to distance R).

Example;

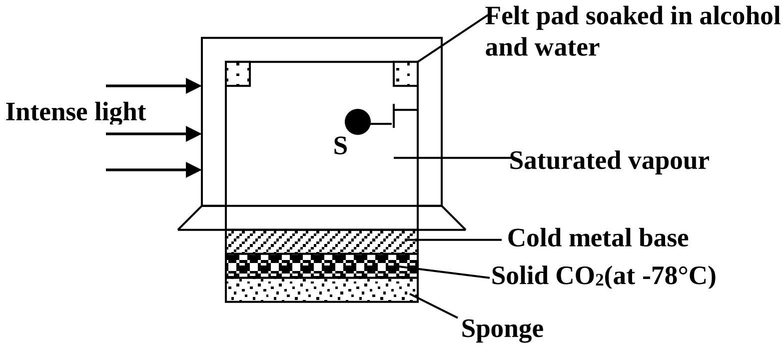
A G.M tube is placed 10cm away from 1g of $^{222}R_n$ and gives a count rate of 85counts per second, the entry window of the G.M tube has the area of 5cm^2 and the background count rate is 10counts/second, calculate;

The activity of the sample and the half life of R_a (ans; A=1.885x10^{4}counts/s $T_{1/2}$ = 10years)

Cloud Chamber

There two types:

*(1) **Diffusion type chamber***

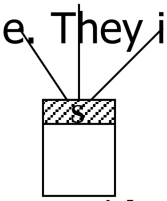


Mode of operation

Alcohol vaporised in the warm upper part of the chamber. It diffuses towards the cold part of the chamber. Above the cold metal base, there will be a layer of air super saturated with alcohol and water vapour.

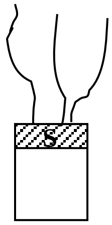
The shield surrounding the radioactive source is removed. The radiation from S ionises the air molecules. The ions provide nuclei for condensation and their paths are seen by means of the intense light directed in the chamber. The tracks of the ions can be photographed. the chamber is cleared of ions by supplying a p.d between the top and bottom of the chamber.

The type of radiation emitted by the radioactive source can be deducted from the tracks formed. α particles proceed without deviation except at the end of their range. They ionise copiously and have well defined range.



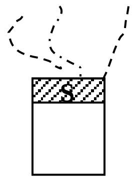
α-particles

Β-particles proceed along tortuous tracks because they are light and are easily deflected by collision with atoms.



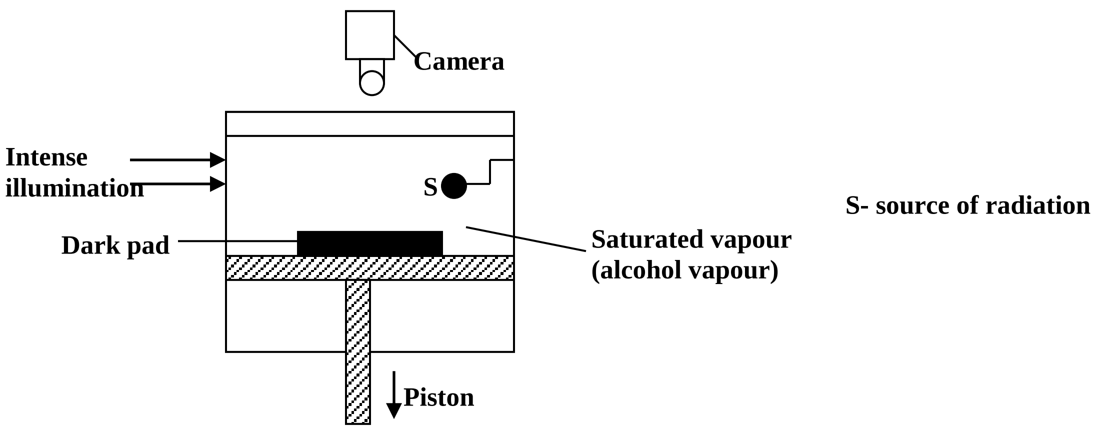
β-particles

γ-rays give rise to diffuse ionisation.



γ-rays

(2) Expansion cloud chamber



Mode of operation

The air inside the chamber undergoes adiabatic expansion by pumping on it. The air cools down as a result. After a few adiabatic expansion, condensation takes on the residual ions (or dust nuclei). The chamber is cleared of these ions by application of a p.d between the top and bottom of the chamber.

The gas in the chamber is then subjected to a precise adiabatic expansion so that the gas becomes super saturated. The shield S is removed, condensation takes place on the ions formed radiation emitted by S. The tracks of emissions of S are photographed and emissions identified.

Example

A source of $\propto$ particles has an initial activity of 2 x 10^5 disintegrations per second. When the $\propto$ particles enter an ionisation chamber, a saturation current of 2 x 10^{-7} A is obtained. If the energy required to produce an ion pair is 32ev. Find the energy of one $\propto$-particle

$$I = ne$$

$$n = \frac{I}{e} = \frac{2\times10^{-7}}{1.6\times10^{-19}} = 1.25\times10^{12}$$

Energy lost per second = $1.25\times10^{12}\times32 = 4\times10^{13}eV$

Energy of one α- particle = $\frac{4\times10^{13}}{2\times10^{5}} = 2\times10^{7}eV$

NUCLEAR ENERGY

Einstein's mass- energy relation

If the mass of the closed system changes by an amount of m, the energy of the system changes by an amount, E = mc^2, where c is the speed of light in a vacuum. The above relation is Einstein's mass- energy relation.

For a given mass, there is energy released.

Recall that 1U = 1.66×10^{-27}kg.

If the mass changes by 1U, then the energy changes is

$$E = mc^2 = 1.66\times10^{-27}\times\left(3\times10^8\right)^2 = 1.494\times10^{-10}\,J = 934MeV$$

Binding Energy

The protons and neutrons of an atom are called nucleons. The energy needed to take all the nucleus a part so that they are completely separated is called the binding energy of the nucleus.

Hence from Einstein's mass- energy relation, it follows that the mass of the individual nucleons is greater than that of the nucleus in which they are together. The difference in mass is a measure of binding energy.

Example

Find the binding energy of a helium nucleus ${}_{2}^{4}He$

Mass of ${}_{2}^{4}He$ = 4.0015U

Mass of ${}_{0}^{1}n$ = 1.0087U

Mass of ${}_{1}^{1}p$ = 1.0073U

$${}_{2}^{4}He \rightarrow 2{}_{0}^{1}n + 2{}_{1}^{1}p$$

mass on the right hand side = $(2\times1.0087)+(2\times1.0073)=4.032U$

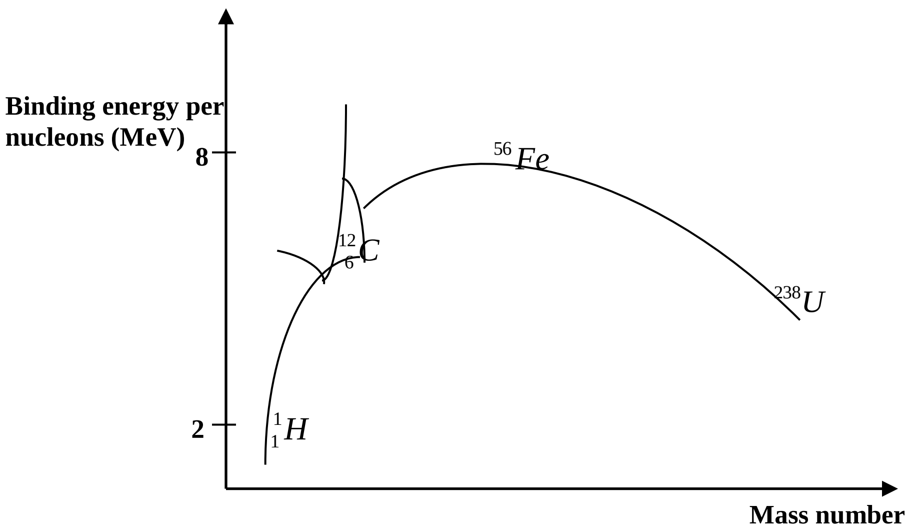
Change in mass, m = $4.032-4.0015=0.0305U$

But 1U = 931MeV

Hence binding energy = 931x0.0305 = 28.4MeV

Binding energy per nucleons is the ratio of the binding energy to the atomic mass of the nucleus.

The binding energy per nucleons of elements of the periodic table varies with mass as shown below.



The higher the binding energy per nucleon, the more stable the nucleus. Excluding the nuclei lighter than ^{12}C, the graph indicates that the average binding energy per nucleon is fairly constant for a great majority of nuclei. The average value is about 8MeV per nucleon. The pitch occur at approximately the ^{56}Fe nucleus which is therefore one of the most stable nuclei.

Nuclear fusion and fission

Nuclear fission

A nuclear fission reaction involves bombarding of the heavy nucleus with a highly energetic particles such as neutrons, protons, deuterons and alpha particles. The heavy nucleus splits into lighter nuclei of higher binding energy per nucleon. The mass deficiency which results is accounted for by the energy released in accordance to Einstein's mass-energy relation. In most nuclear fission reactions, neutrons are used to induce a reaction because of being neutral, they can penetrate the nucleus.

When ^{235}U splits, it produces nuclei that are lighter and hence have higher binding energy.

Examples of nuclear fission

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{90}_{38}Sr + {}^{136}_{54}Xe + 10{}^{1}_{0}n$$

Find the energy released by 1kg.

Mass of ^{235}U = 235.0439U, Mass of ^{90}Sr = 89.9073U,

Mass of ^{1}n = 1.0087U, Mass of ^{136}Xe = 135.907U

Mass on left hand side = 235.0439+1.0087 = 236.0526U

Mass on right hand side = 89.9073 + 135.907 + 10x1.0087 = 235.9013U

Change of mass, m = 236.0526 - 235.9013 = 0.1513U

Energy released by a nucleon of U235 = 0.1513x931Mev = 140.8603MeV

Energy released by 1kgof ^{235}U = $\frac{1000}{235} \times 6.025 \times 10^{23} \times 140.8603 = 3.61 \times 10^{26}\, MeV$

In the above example, when the emitted neutrons encounter with other Uranium nuclides, they bombard the uranium and more splitting occurs with the release of more energy. The produced neutrons are called fission neutrons, and when this occurs, the reaction is called a chain reaction. In a chain reaction, a lot of energy is produced and unless this energy is controlled, the reactions may cause an explosion. Chain reaction is applied in making nuclear bombs.

Exercise

238U disintegrates by emission of an α- particle according to the equation

$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$

calculate (i) the total energy released in the disintegration (4.2315MeV)

(ii) the kinetic energy of the alpha particles, with the nucleus being at rest before disintegration.(4.16MeV)

Mass of ^{238}U = 238.1249U, Mass of ^{4}He = 4.00387U,

Mass of ^{234}Th = 234.11650U, 1U = 930MeV

Nuclear fusion

A lot of energy is released when the nuclei of lighter elements fuse together to form a heavy nucleus. The fusing together of nuclei to form a heavy nucleus is called nuclear fusion.

Example

Formation of alpha particles when lithium fuses with hydrogen.

$^{7}_{3}Li + ^{1}_{1}H \rightarrow ^{8}_{4}Be \rightarrow 2^{4}_{2}He$

Mass of ^{7}Li = 7.0160U, mass of ^{1}H = 1.0078U,

Mass of ^{4}He = 4.0026U, 1U = 931MeV

solution

Mass on left hand side = 7.0160 + 1.0078 = 8.0238U

Mass on right hand side = 2x4.0026 = 8.0052U

Change of mass = 8.0238 – 8.0052 = 0.0186U

Energy released = 0.0186x931MeV = 17.317MeV

Energy released by 1kg of the reactants = $\frac{1000}{8} \times 6.025 \times 10^{23} \times 17.317 = 1.304 \times 10^{27}\ MeV$

Exercise:

Calculate the energy released by the reactant of two deuterium fusing to form helium according to the equation. $2^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$

Mass of $^{2}_{1}H = 2.01421U,$

Mass of ${}_{1}^{3}He = 3.0160U,$

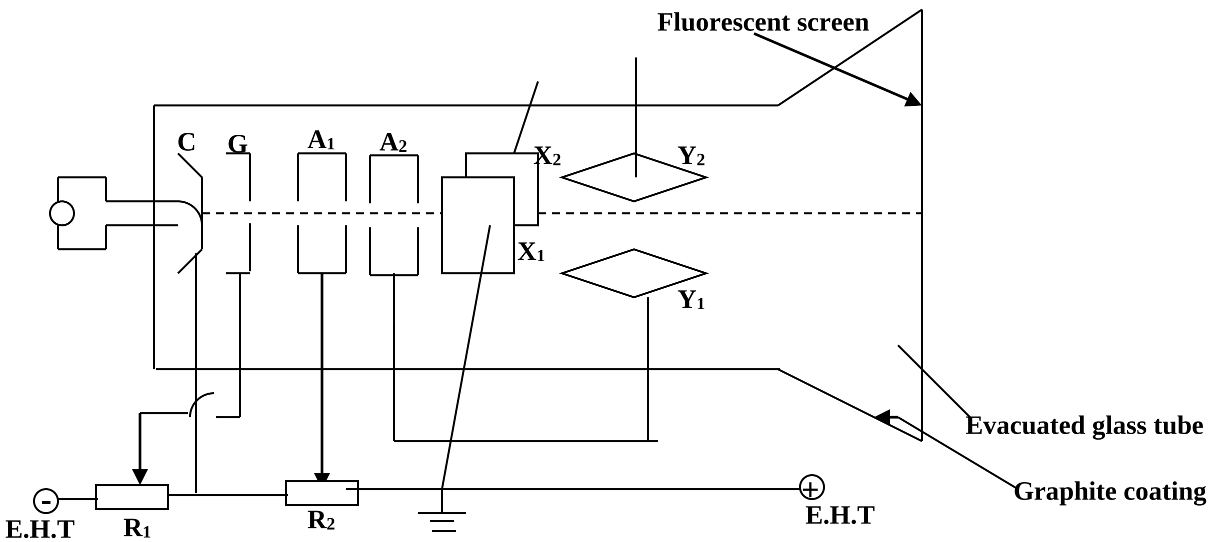
Mass of ${}_{0}^{1}n = 1.0087U,$

1U = 931MeV

The sun contains a considerable amount of hydrogen. It is believed that the energy of the sun is due to nuclear fusion of the hydrogen atoms. Fusion is capable if the nuclei concerned are able to approach each other close enough and if the temperatures are very high. These conditions are achieved in the sun.

Electronic Devices

1. Cathode ray Oscilliscope



Uses of the parts

Evacuated Glass tube:- the glass tube evacuated to prevent scattering of the electron beam when electrons collide with air molecules.

Indirectly heated Cathode, ***C*** :-Emits electrons by thermionic emission.

The grid, ***G***:- it consists of a hollow metal cylinder with a small hole at the end. It is held at variable negative potential relative to the cathode by means of the potential divider R_1, The grid serves two purposes namely: (i) as a brightness control

(ii) it refocuses the electron beam so that the beam emerges from the hole as a narrow beam.

Anodes ***A_1*** *and* ***A_2:-*** These are held at a positive potential relative to the cathode. The anode accelerates the electron beam along the tube and also focuses the electron beam by means of the potential divider R_2.

X-plates, ***X_1*** *and* ***X_2***:-these are vertical plates but they deflect the beam horizontally when a p.d is applied across them.

Y-Plates, ***Y1*** *and* ***Y2***:- These are horizontal pates but deflect the beam vertically when a p.d is applied across them.

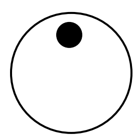
Fluorescent Screen:- This is coated with fluorescent material such as Zns. It enters light when struck by electron beam.

Graphite Coating:- Enables light to be seen only on the screen as the graphite coating absorbs the electron's kinetic energy.

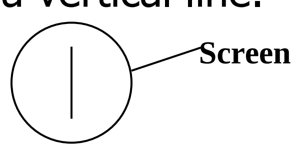
*Power supply:-*This is a smoothened rectified a.c, fed through a chain of resistors.

Operation of the CRO

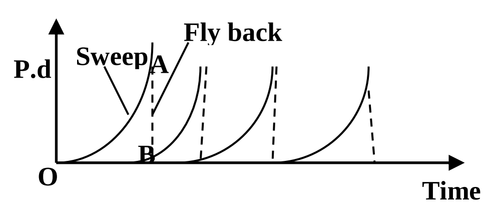
Suppose the X- plates were shunted and a d.c voltage was applied to the Y-plates. The electron spot would be deflected vertically.



If the X- plates are shunted and an a.c is applied to Y-plates, the electron beam is drawn into a vertical line.

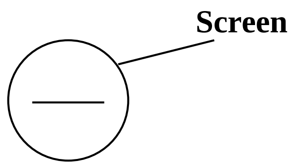


To observe the waveform of the a.c signal applied to the Y-plates, a special voltage called time base connected to the X- plates. The time base has a saw-tooth waveform and is generated by a special in the CRO. The saw tooth voltage which sweeps the electron beam from left to right at a constant speed.

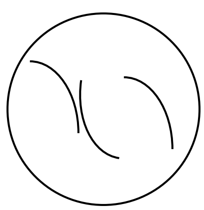


The time taken for p.d to fall from A to B known as the fly back time, is extremely small compared to the time taken to rise from O to A. Hence the time taken by return of the electron beam to the original position at the other end of the screen is small.

When no signal is applied to the Y- plates, the voltage V, causes the electron beam to sweep horizontally to and fro on the screen as shown.



To observe the wave form of the applied voltage to Y-plates, the frequency of the time base is synchronised with the frequency of the signal applied to Y-plates. For an a.c signal applied across the Y- plates and time base on the X-plates, one observes the waveform shown below;



Uses of a CRO

a) *Displaying of waveforms*: the amplitude and frequency of the wave can be obtained.

b) *Measurement of Voltage*: An unknown voltage is applied across the Y-plates. If the time base is switched off, a vertical line is obtained on the screen. This can be centred and its length measured. This is proportional to twice to the amplitude or peak voltage, V_0.

c) *Comparison of frequencies of two waveforms*: suppose two waveforms of frequency f_1 and frequency f_2 appear on the screen of the CRO having two Y-inputs or are displayed at a time on the CRO with a single Y-input. If x_1 and x_2 are distances occupied by one cycle for two waveform, then the ratio $\frac{f_1}{f_2} = \frac{T_2}{T_1} = \frac{x_2}{x_1}$, where T_1 and T_2 are the periodic times of the two waves respectively.

d) *Measurement of phase difference using a double beam CRO*: the two waveforms to be compared have the same frequency. Suppose they are displayed simultaneously by applying them to the two Y-input.

Comparison o CRO with a moving coil Voltmeter.

a) The CRO has very high impedence. It gives accurate voltages than a moving coil voltmeter.
b) A CRO can measure both d.c and a.c voltage. A moving coil voltmeter measures only D.C voltages unless a rectifier is used. The CRO gives a peak to peak values of a.c.
c) A CRO has negligible inertia as compared to a moving coil voltmeter. The CRO respond almost instantaneously.
d) CRO doesn't give direct voltage readings.

Question

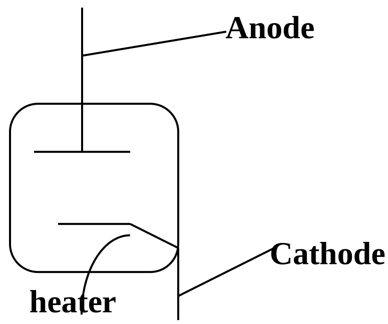
A CRO has its Y- sensitivity set to $10Vcm^{-1}$. A sinusoidal input is suitably applied to give a steady trace with the time base set so that the electron beam takes 0.01s so traverse the screen. If the trace seen has a total peak to peak height of 4cm and contains two complete cycles, what is the r.m.s voltage and frequency of the input? (14.1V, 200Hz)

2. Thermionic Diode

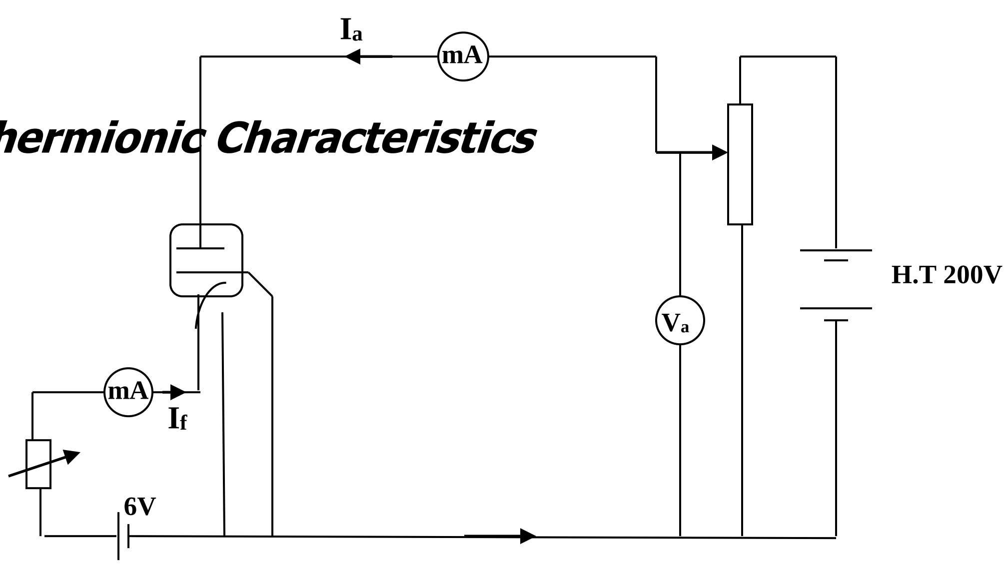
Structure

It consists of an anode usually in form of a nickel cylinder which surrounds the cathode in an evacuated glass bulb. In the indirectly heated cathode type, the cathode is a nickel tube with a tungsten filament (or heater) inside it. The heater is insulated electrically from the cathode by packing alumina inside the nickel tube. The outside of the tube is coated with a mixture of Barium and Strontium oxides. The mixture has a low work function(about 1.8eV) and emits electrons at relatively low temperatures (about 1100K)

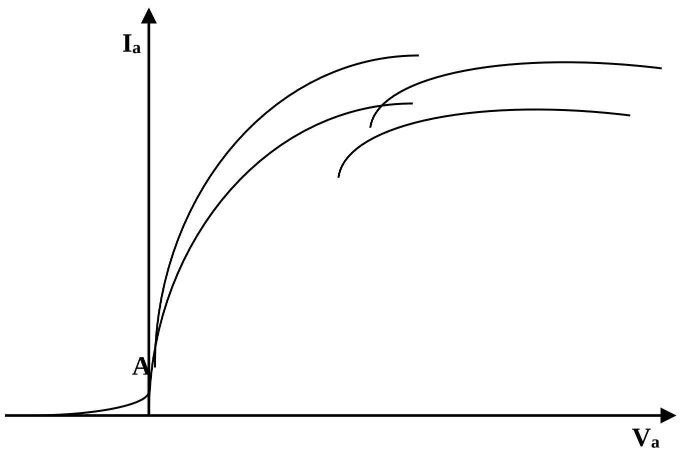
Symbol of a diode



Thermionic Characteristics



Keeping the filament current If constant, the p.d V_a between the cathode and the anode is varied. The corresponding anode current I_a is measured. A graph of I_a against V_a constitutes the anode – current anode voltage characteristics. By setting the filament current to other constant values, the corresponding I_a-V_a characteristics can be obtained. These features can be shown below



For Va = 0, electrons are emitted by the cathode with a range of speeds. A few of the electrons are emitted with sufficient kinetic energy to be able to reach the anode. This leads to a small current. If the anode is made negative relative to the cathode, a reverse current exists for negative potentials up to about 0.5V and then decreases to zero.

Region AB: Here V_a is small. Only those electrons emitted with high speeds will be able to reach the anode. The majority of the electrons are emitted with low kinetic energies and are repelled back towards the cathode. The electron distribution around the cathode constitutes a negative space charge. The current I_a is small.

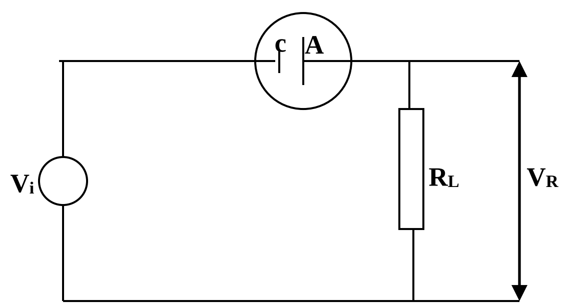
Region BC: as V_a increases the attraction of the space charge by the anode increases. This results in a larger anode current. This region is called *space charge limited region*.

Region CD: the anode voltage V_a is so large that all the electrons emitted per second by the cathode reach the anode. The space charge is overcome. A constant current, called saturation current flows. Region CD is also called the *temperature limited region* because when the temperature of the cathode increases, more electrons are emitted per second by the cathode. A higher saturation current therefore flows.

Applications of the thermionic diode

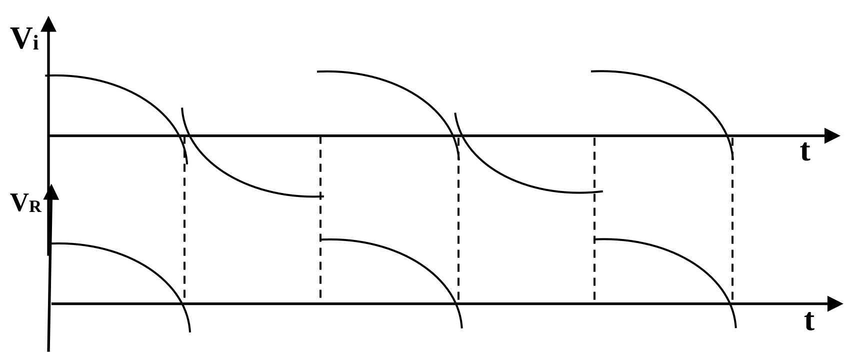
(a) Half- wave rectification

Suppose a thermionic diode is connected in series with a source of alternating voltage V_i and a load R_L



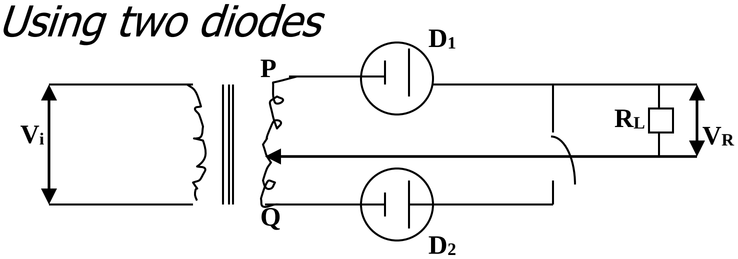
During the half cycles when A is positive relative to C the diode conducts and a p.d V_R appears across the load R_L. During the half cycles when A is at a negative potential relative to C, the diode does not conduct and no p.d appears across R_L. The a.c is half-wave rectified.

The input and output voltage wave forms are compared in the diagram below.



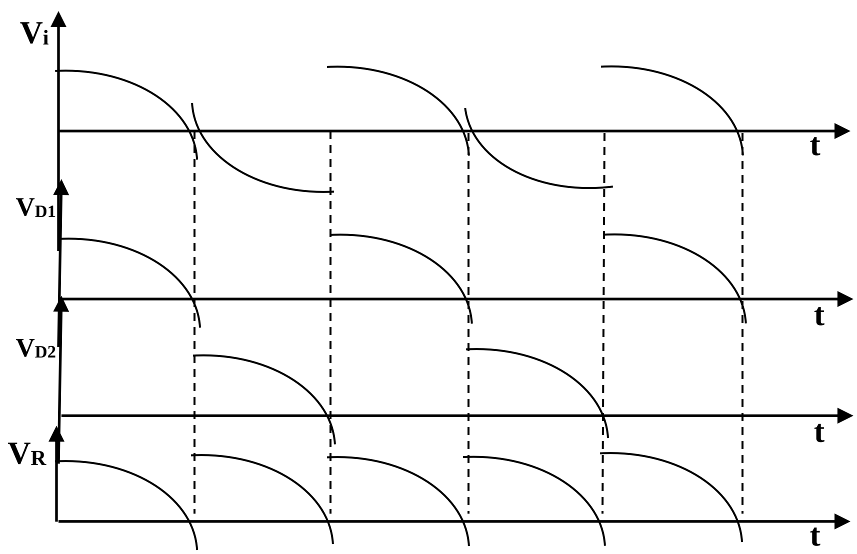
(b) ***Full wave rectification***

(i*) Using two diodes*



When P is at negative potential relative to Q, diode D_1 conducts whereas D_2 doesn't.

When P is at a positive potential relative to Q, diode D_2 conducts whereas D_1 does not. Current flows in the same direction through the load R_L during both positive and negative cycles of the input voltage V_i.

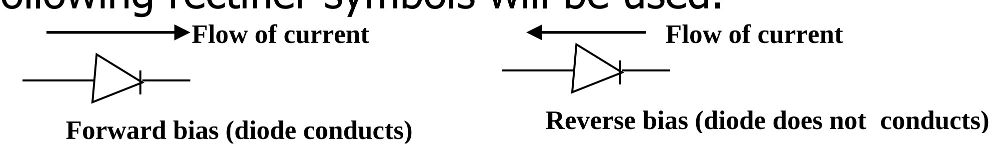


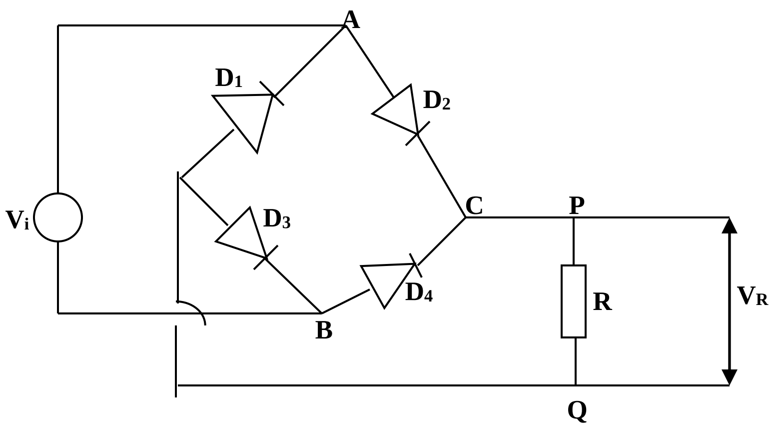
V_{D1} and V_{D2} is output pd due to conduction of diode D_1 and D_2 respectively.

V_R is the output voltage across load R_L.

(ii) *Using four diodes*

The following rectifier symbols will be used.

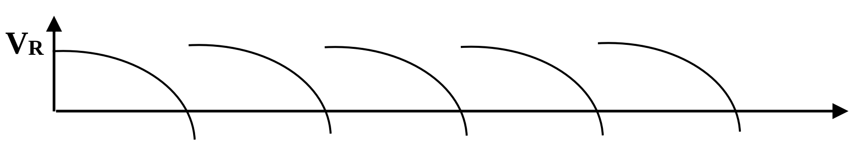




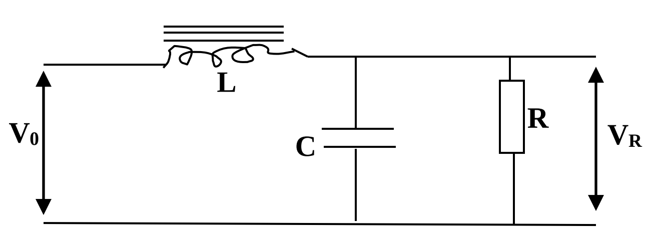
During the half cycles when A is at positive potential relative to B, diodes D_2 and D_3 are forward biased hence they conduct and current flows through the resistor R in the direction P to Q. Diodes D_1 and D_4 are reverse biased and they do not conduct.

During the half cycles when B is at positive potential relative to A, diodes D_1 and D_4 are forward biased and they conduct. Currents flows through resistor R in the direction P to Q. Diodes D_2 and D_3 are reverse biased and don't conduct.

The voltage cross R will have the form:

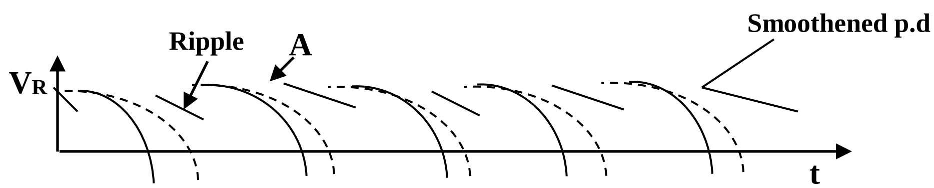


The output voltage can be smoothened by using filter circuits of the form shown below:



The back emf induced in the inductor by the fluctuating voltage opposes the voltage fluctuations. The capacitor acts as a reservoir to steady the remaining voltage fluctuations.

The voltage across the resistor R has the form shown:



At points such as A, the p.d across the load has just reached its maximum value. If the capacitor was not present, the p.d would start to fall to zero along the broken curve. However, as soon as the p.d across the load starts to fall, it becomes less than that across the capacitor and the capacitor starts to discharge through the load.