S.6 STAHIZA RECESS TERM WORK 2020

VECTORS

1. A line passes through the point with position vector 4**i** + 7**j**, + 5**k** and is parallel to the vector 3**i** – 2**j** + 4**k**. Find the
   (i) Equation of the line
   (ii) Point of intersection of the line in (i) and the plane x + 2y – z = 4
2. (a) If N is the foot of a perpendicular from point M(5, – 3, 2) to the line
   $\frac{x+1}{2} = \frac{y-4}{1} - \frac{Z-2}{-1}$
   Determine the coordinates of N.
   (b) Find the equation of the plane through points M, N and P (– 1, 4, 2) on the line. Hence determine the angle MPN

3. (a) Show that the points ***A (6, –1, 8)*** , ***B(0, 7, 3)*** and ***C (2, 1, 5)*** are vertices of triangle ***ABC***.

(b) Find the perpendicular distance from the point ***P (4, 6, –4)*** to the line passing through the points ***A (2, 2, 1)*** and ***B (4, 3, –1)***

4. (a) (i) Show that the points (1, 2, 3), (3, 8, 1) and $(7, 20, -3\ )$ are collinear. (02 marks)
   (ii) Coordinates A $(3,8,1) and\ B(7,20,-3)$ are externally bisected from point B by Point M, find coordinates of M.
   (b) Find the point of intersection of the lines, $\boldsymbol{r} = 2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + 5\boldsymbol{k} + \lambda(-\hat{\boldsymbol{\imath}} - 2\boldsymbol{k})$ and
   $\frac{x-1}{1} = \frac{y-2}{0} = \frac{z-1}{3}$

(c) Find the shortest distance of $P(3, +1, -1)$ from the line
$\boldsymbol{r} = (1 - \lambda)\hat{\imath} + (2\lambda - 1)\hat{\boldsymbol{\jmath}} + +(2\lambda - 2)\hat{\boldsymbol{k}}$

5. (a). Given $a = i + 2j + 3k$ and $b = 4i - j + 2k$ are vectors, find a vector which is perpendicular to both a and b

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(b). The point A has coordinates $(2, 0, {}^{-}1)$ and the plane π has equation $x+2y-2z=8$.
The line through A parallel to the line $\frac{x}{{}^{-}2} = y = \frac{z+1}{2}$ meets π at the point B and the perpendicular from A to π meets π at the point C.
(i). Find the coordinates of B and C.
(ii).Show that the length of AC is $\frac{4}{3}$

6. a) Find the angles between the vectors $a = 2i + 3j - k$ and

$b = 5i + 2j + k.$

b) A plane has the points $A(2, \quad 1, \quad 3)$, $B(0, \quad -6, \quad 2)$ and $C(3, \quad 2, \quad 1)$ on it. Determine the Cartesian equation of the plane.

c) The normal to the plane in (b) above is a directional vector to the line passing through $(1, \quad 1, \quad 5)$. Find in Cartesian form, the equation of the line.

7. $A(2, -1, 4)$, $B(3, 0, 2)$, $T(4, 2, -3)$ and $S(0, -3, 8)$. are points on the same plane Find;

(a) Point of intersection of lines AB and TS.

(b) Equation of the plane containing points A, B, T and S.

8. a) Show that a point whose position vector is $\hat{i} - 9\hat{j} + \check{k}$ lies on the line with vector equation $r = 3\hat{i} + 3\hat{j} + -\check{k} + \lambda(\hat{i} + 6\hat{j} - \check{k})$.

ii) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$.

b) O, A and B are non-collinear points such that $\overrightarrow{OA}$ = **a** , $\overrightarrow{OB}$ = **b** and C is the midpoint of $\overrightarrow{AB}$. D is a point on OB such that 4 $\overrightarrow{OD}$= $\overrightarrow{OB}$, T is the point of intersection of OC and AD. Find vector of $\overrightarrow{OT}$ in terms of **a** and **b**. (6 mks)

9.. (a) Determine the coordinates of the point of intersection of the line. $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{5}$ and the plane $x + y + z = 12$.

b Find the angle between the line $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$ and the plane $x + y + z = 12$.

10. (a) (i) A line joining point A (3, -2, 5) and B (9, 2, -1) is divided by point C externally in the ratio 3:5, Find the position vector of point C.

(ii) Given that the point C divides the line AB in the ratio 1:2 and the position vectors of A and C are $-4i - 3j + 5k$ and $3i - 2j + 12k$ respectively. Find the coordinates of point B.

(iii) Find the angle between $\frac{x-3}{2} = \frac{2y-1}{4} = \frac{3z-1}{4}$ and $-4x + 3y + 2z = 7$

11. Find the angle between the line of intersection of the planes 2x + y + 3z = 4 and 3x + 2y + 2z = 7 and the line $\frac{1-x}{1} = \frac{y-2}{2} = \frac{z-3}{4}$

12. Find the distance between the planes 2x – 3y + 4z = 7 and

8x – 12y + 16z = 6.

13. Find the equation of the plane through the (1,0,-1) and containing the

line $X = -y = \frac{z}{2}$.

14. Find the equation of the plane containing the line

$$\frac{x-3}{5} = \frac{y+1}{2} = \frac{z-3}{1} \ and \ \frac{3-x}{-2} = \frac{y+1}{4} = \frac{z-3}{3}.$$

15. Find the equation of the plane through the point (1,-2,1) which also Contains the line of intersection of the planes $x + y + z + 6 = 0$ and $x - y + z + 5 = 0$.

16. (i) Show that vectors $i - 2k, -2i + j + 3k$ and $-i + j + k$ form a triangle .

Hence or otherwise find the area of the triangle.

(ii) Show that these lines are skew , $\frac{x+3}{0} = \frac{y+2}{-2} = \frac{z-4}{2}$ and $\frac{x-7}{-1} = \frac{y}{2} = \frac{z+5}{1}$

17. Show that the vectors **a=3i+j-4k,b=4i-j-3k,c=5i-3j-2k** are coplanar.

18. The equations $\frac{x-3}{2} = \frac{y-5}{1} = \frac{z-2}{-3} \ and \ \frac{x+1}{3} = \frac{y+4}{1} = \frac{z-2}{-2}$ represent pipes A and B in a chemical plant. Find the the length of the shortest pipe that can be fitted at end points of A and B.

19. Giiven the points A (2,-4,7),B (1,3,5) and C (2,-3,1).A line through points A and B is perpendicular to a plane containing point C. Find the equation of the plane,hence the point of intersection of the line and the plane.

CONTINUATION OF DIFFERENTIAL EQUATIONS (DE'S)

20. (a) Solve the differential equation

$\frac{dy}{dx} = x - \frac{2y}{x}$ Given y (2) = 4

(b) A plague wipes out a community at a rate proportional to the population. If the original population is 4 million, and the population reduces from 2.5 to $^1/_5$ million in 5 years

Find how long it takes to reduce the original population to 1 million

21. (a) Solve the differential equation: $\frac{dy}{dx} - ytanx = cos^2x$

(b) Given that $y = e^{tanx}$. Show that $\frac{d^2y}{dx^2} - (2tanx + sec^2x)\frac{dy}{dx} = 0$

22. (a) Solve the differential equation.

$x\frac{dy}{dx} = 3x - 2y$ for $x > 0$, given that $y = \frac{3}{4}$ when $x = 1$.

(b) On a local poultry farm, the rate at which the birds are decreasing due to a certain disease is proportional to the square of the number of birds present. Initially there were 600 birds and after 10 days there was 500 birds.

(i) Form a different equation relating number of birds x after time t days. And

solve it.

(ii) find when there will be only 300 birds on the farm.

23. A hunter killed a loin and recorded the temperature of the body of the lion. Where he noticed that the body originally at 38^0c was cooling in accordance with Newton's law of cooling. After 2hrs the temperature of the body was 34^0c and the temperature of the surrounding air was constant 20^0c.

a) Find the temperature, θ, of the body as a function of t , the time in hours since the lion was killed.

b) If at 5:00pm, the temperature of the body was 30^0c, find the time when the lion was killed.

24. At 3:00pm, the temperature of a hot metal was 80^0C and that of the surrounding 20^0C. At 3:03pm, the temperature of the metal had dropped to 42^0C. The rate of cooling of the metal was directly proportional to the difference between its temperature θ and that of the surroundings.

(i) Write a differential equation to represent the rate of cooling of the metal.

(ii) Solve the differential equation using the given condition.

Iii Find the temperature of the metal at 3:05pm.

25. A pan of water at 65^0C is standing in a kitchen where temperature is a steady 15^0C. Show that after cooling for t minutes, the water temperature צ can be modeled by the equation.

צ = 15 + $50e^{-kt}$

i Given that after 10 minutes, the temperature of water has fallen to 50^0C, find the value of k.

ii Find the temperature after 15 minutes.

26 . a) Solve the differential equation, $x\frac{dy}{dx} = y + kx^2 cosx$ given that $y = 2\pi$ when $x = \pi$.

b) A certain chemical reaction is such that the rate of transformation of the reacting substance is proportional to its concentration. If initially the concentration of the reagent was $9.5gm$ per litre and if after 5 minutes the concentration was $3.5gm$ per litre, find what the concentration was after 2 minutes.

27. The rate at which the population of the town increases is directly proportional to ***80 %*** of the population at that time. Initially the population was ***150,000*** and then ***5*** years later the population was found to be ***300,000,***

(a) Write down a differential equation representing the above information and solve it.

(b) Find**:**

(i) the population of the town by the tenth year

(ii) how long it will take for the population to be ***270,000***

28 (a). Find the particular solution of the differential equation

$\frac{dy}{dx} + 3y = 2\,e^{-3x} Sin2x$ Given that y (0) = 4

(b) . A plague wipes out a community at a rate proportional to the population. If the Original population is 4 million, and the population reduces from 2.5 to $\frac{1}{5}$ million in 5years, find how long it takes to reduce the original population to 1 million

29. A liquid is being heated in an oven maintained at constant temperature of 180°C. It is assumed that the rate of increase in temperature of the liquid is proportional to (180 – θ), where θ°C is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from 0°C to 120°C in 5 minutes, find the temperature of the liquid after a further 5 minutes.

30. The rate of speed of bush fire spreading is proportional to the area of unburnt Bush .At a certain moment 0.6 of the bush had been burnt, 2hrs later 0.65 of the bush was bunrt.Find the fraction remaining unburnt after 5hours.

31. A man starts to climb a mountain whose height is 1000m above its foot. He notices that the rate at which the temperature drops with height raised is directly proportional to the height. The temperature is 16^0c at the foot and drops to -9^0c at the top of the mountain. Find the height at which the temperature reaches the freezing point of water.

<u>INEQUALITIES AND FURTHER CURVE SKETCHING</u>

32 Solve the inequality,

(i) $\frac{x-1}{x-2} > \frac{x-2}{x+3}$ (ii) $2|x-1| < |x+3|$ (iii) $|5x-6| < x^2$

33. (i) Sketch the curve $f(x) = \frac{x^2+2x+3}{x^2+3x+2}$. (ii) $f(x) = \frac{9}{x+2} - \frac{1}{x}$

(iii) Show that for real x, this range $-1 < \frac{2x+5}{x^2-4} < -1/4$ cannot take place.

34. If y =$\frac{(x-1)^2}{(x+1)(x-3)}$. Find the values of y for which the curve is not defined,

Hence find the nature of the turning points. sketch the curve.

35 Find the co-ordinate of intersection of y =$\frac{x}{x^2+1}$ and y= $\frac{x}{x+3}$.

Sketch both curves on Same axes and show that the area of finite region in the first quadrant enclosed by two curves is $\frac{7}{2}ln5$ -3ln3-2.

36 The domain of the function defined by f(x)= $\frac{4(2x-7)}{(x-3)(x+1)}$ is the set of all real Values of x other than 3 and -1.Express f(x) in partial fractions,hence or Otherwise,show that $f^1(x)$ is zero for two +ve integral values of x.Find the

Turning points of this function.Hence sketch f(x).Dertermine the area of the

Region bounded by the cyrve,the x-axis and the lines $x = 4, x = 6$.

(Leave your answer in logarithmic form.)

37 (a) Find the equations of the tangents to the ellipse $3x^2 + 4y^2 = 12$

which are parallel to the chord x + y = 1

(b) A point P on the curve is given parametrically by x = 3 – Cosθ and y = 2+ Secθ

Find the;

(i) Equation of the normal to the curve at the point $\theta = \pi/3$

(ii) Cartesian equation of the curve

CONIC SECTIONS

38. Show that at any point on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ may be written as (a sec θ, b tan θ)

39. Find c in terms of a, b , m if y = m x + c is a tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ .Find the asymptotes.

40. The normal at the point P(5 Cosθ,4sinθ) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the

X and Y axes at Land M respectively. Show that the locus of R ,the midpoint of

LM is an ellipse having the same eccentricity as the given ellipse.

41, The curve $b^2x^2 + a^2y^2 = a^2b^2$ intersects the positive X-axis at A and the Y-axis at B.

(i) Determine the equation of the perpendicular bisector of AB.

(ii) Given that this line intersects the X-axis at P and that M is the bisection of AB.Show that the area of triangle PMA is $\frac{b(a^2+b^2)}{8a}$.

42 State the vertex,focus and directix of **$8y^2 + 6y - 9 = 4x$** and **$2x^2 + 3y^2 - 4x + 9y + 5 = 0$.**

43. Show that the two tangents of gradients m to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{(a^2m^2 + b^2)}$.find eqnsof tangents to ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ at (-2,5).

44. If the line $cy + x + d = 0$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; then show that $d^2 = a^2 + b^2c^2$. Hence determine the equations of the four common tangents to the ellipse $4x^2 + 14y^2 = 56$ and $3x^2 + 23y^2 = 69$.

45 . (a) Given the equation $y = 5x - 2x^2$

(i). Show that the equation represents a parabola and find the length of its latus rectum.

(ii). Find the co-ordinates of the focus and the equation of the directrix.

(b). A conic section is given by $x = 4\text{Cos}\theta$, $y = 3\text{Sin}\theta$. Show that the conic section is an ellipse and determine its eccentricity**.**

46. a) Find the Cartesian equation of a curve whose polar equation is given by $r = atan\theta$.

b) Obtain the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(acos\theta, asin\theta)$. If the tangent cuts the x and y axes at points Q and R respectively, determine the locus of the midpoint of QR.

END

- ***To the <u>PROBLEMS</u> of your life,you're the <u>SOLUTION</u> ,and to the <u>QUESTIONS</u> of your life ,you're the <u>ANSWER.</u>***
- ***If you are going to achieve excellence in big things, you develop the habit in little matter***
- ***Failure defeats only LOSERS but it inspires WINNERS.***
- ***He who thinks that he can make it,makes it.***

FINALLY, .. " SUCCESS COMES TO A PREPARED MIND"....

<u>MJ@2020</u>