S.6 P425/1 QUESTIONS 2020 (STAHIZA)

1. (a) Prove that the number $a_n = 4^n + 5$ is divisible by 3 for all positive integral values of n.

(b) The sum of **n** terms of the progression in $\mathbf{S_n = 2n^2+3}$.Find the tenth term.

(c) Prove by induction that.

(i) $\sum_{r=1}^{n} 3^{n-1} = \frac{1}{2}(3^n-1)$

(ii) $\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)} = \frac{n}{3(2n+3)}$

iii) $\sum_{r=1}^{n} Sin\, r\theta = \frac{Sin(\frac{n+1}{2})\theta Sin(\frac{n\theta}{2})}{Sin\frac{\theta}{2}}$

(d) DON operate an account with Cairo bank which pays a compound interest rate of 12.5% per annum. They opened the account at the beginning of the year with Shs800,000 and deposit the same amount of money at the beginning of every year. Calculate how much they will receive at the end of 10 years. After how long will the money have accumulated to 3.32 million.

2. (a) A polynomial P(x) is a multiple of x – 3 and the remainders when P(x) is divided by x + 2 and x-5 are 6 and-7 resp. Find the remainder when P(x) is divided by $x^3 - 5x^2 - 4x + 20$.

(b) When $P(x) = x^3 + ax^2 + bx + c$ is divided by $x^2 - 4$, the remainder is 2x + 11. Given that x + 1 is a factor of P(x), Find the values of a, b and c.

(c) If the function $P(x) = x^4+ax^3+bx^2+6x-5$ is divisible by $(x-2)^2$.

Find the values of a and b.

(d) Give that $x^4 - 6x^3 + 10x^2 + ax + b$ is a perfect square, find a and b.

The roots of the equation $ax^2+bx+c=0$ where a,b and c are non Zero constants are α and β,and the roots of equation $ax^2+2bx+c=0$ are θ and μ. Find the equation whose roots are $\alpha\theta+\beta\mu$ and $\alpha\mu+\beta\theta$.

3. (a) Express in partial fractions; $f(x) = \frac{x^2-x}{(x^2+3)(x^2+2)}$ Hence $\int f(x)dx$

(b) $\int \frac{x^6-x^5-4x^2+x}{x^4+3x^2+2} dx$

(c) Express $f(x) = \frac{4x+5}{(x+1)(2x+3)}$ in partial fractions; hence find $f'(x)$ and $f''(x)$.

d) Express $y=\frac{2x^2+3x+5}{(x+1)(x^2+3)}$ in partial fractions and hence show that $\frac{dy}{dx}=-\frac{2}{3}$ when x=0 ,and evaluate $\int_0^{\sqrt{3}} ydx$. state the mean value of y in the interval $0\leq x\leq\sqrt{3}$

4. Differentiate;

(a) $\sqrt[3]{\frac{x^2-2}{x^2+4}}$ (b) $\sqrt{\frac{Sin\ 2x}{Cos^3x}}$ (d) $e^{-3x}sin^2 2xcos3x$ (e) $\frac{e^{2x}tanx}{(x^2-2)sinxcosx}$

(c) (i) If y = t^2 - cos t and x = sin t, find $\frac{d^2y}{dx^2}$. (ii)Find $\frac{dy}{dx}$ of **tan4x⁰** from1st principles.

5. Integrate the following;

(a) $\int_1^3 \frac{2x^2+3x}{2x-1}$dx (b) $\int \frac{x+1}{125x^3-8}$dx (c) $\int \frac{1}{(1+\sqrt{x})\sqrt{x}}$dx

(d) $\int_0^{\Pi/2} Sin^3xCos^2x$ dx (e) $\int_0^{\Pi/2} Sin\ 2x\ Sin\ 5x$ dx

(f) Show that $\int_0^{\Pi/4}(1+tanx)^2$ dx = 1 + $ln2$ (g) $\int \frac{1}{x^4-9}dx$

(g) Show that $\int_0^1 \frac{1}{(x+1)(x^2+2x+2)}$dx = $\frac{1}{2}ln\frac{8}{5}$

(h) Show that $\int_0^{\Pi/2} \frac{Sin^3x}{2-Sin^2x}$dx = $\frac{\Pi}{2}$ - 1

(i) Show that $\int_0^{\Pi/2} \frac{4}{3+5cos\theta}$ dθ = ln 3

(j) $\int_0^{\Pi/4} \frac{1}{5Cos^2\theta-1}$ dθ (k) $\int_0^3 \frac{x+3}{x^2+3}$ dx

(l) $\int xlnx\ dx$ (m) $\int xtan^2 x$ dx (n) $\int x^3\ Sec\ x^2$ dx

(o) $\int \frac{6(x-3)}{x(x^2+9)}$ dx (p) $\int \frac{e^x+e^{2x}}{1+e^{2x}}$dx q) $\int_0^2 \frac{x^4+2x}{(x-1)(x^3-1)}\ dx$

(r) Evaluate $\int_0^{\frac{\pi}{6}} secxdx$ using **sinx=u**.Hence show that $\int_0^{\frac{\pi}{6}} sec^3xdx=\frac{1}{3}+\frac{1}{4}ln3$

6. (a) If y = e^xsinx, Show that $\frac{d^2y}{dx^2}$ = 2($\frac{dy}{dx}$ - y). By further differentiation of this result, find the Maclaurin's expansion as far as the term in x^6 .

(b) Show that using maclaurins theorem,$\ln\sqrt{\frac{1+x}{1-x}}=x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}$.

(c) Show that if y = $e^{3x} sin4x$, $\frac{d^2y}{d^2x} - 6\frac{dy}{dx} + 25y$ =0.

7. (a) Expand $(8 + 3x)^{1/3}$ in ascending powers of x as far as the term in x^3, stating the values of x for which the expansion of x is valid. Hence obtain an approximate value for $\sqrt[3]{8.72}$

(b) Expand $(1 + 16x^2)^{1/2}$ in descending powers of x in the term as far as the third term.

(c) Prove that if x is so small that its cube and other higher powers are neglected,then

$\sqrt{\frac{1+x}{1-x}}$ =1+x+$\frac{1}{2}x^{2.}$ By taking x=$\frac{1}{16}$,Prove that $\sqrt{17}$ =4$\frac{33}{128}$

8 (a) According to Newton's law, the rate of cooling of a body in air is proportional to the difference between the temperature T of the body and T_o of the air. If the air temp is kept constant at 20^0c and the body cools from 100^0Cto 60^0C in 20 minutes, in what further time will the body cool to 30°C.

(b) A liquid is being heated in an oven maintained at constant temperature of 180°C. It is assumed that the rate of increase in temperature of the liquid is proportional to (180 – θ), where θ°C is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from 0°C to 120°C in 5 minutes, find the temperature of the liquid after a further 5 minutes.

(c) The rate of speed of bush fire spreading is proportional to the area of unburnt

Bush .At a certain moment 0.6 of the bush had been burnt, 2hrs later 0.65 of

The bush was bunrt.Find the fraction remaining unburnt after 5hours.

9. (a) (i) If Z_1 = 6(Cos $\frac{5}{12}\Pi$ +i Sin$\frac{5}{12}\Pi$) and Z_2 = 3($Cos\frac{1}{4}\Pi$ +i Sin $\frac{1}{4}\Pi$). Find Z_1Z_2 and $\frac{Z_1}{Z_2}$ in the form $a + ib$.

(ii) Express each of the complex numbers in a polar form if

$Z_1 = \frac{(1-\sqrt{3}\,i)^4\,(2+2\sqrt{3}\,i)^5}{(3i-4)^3}$ and $Z_2 = \left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)^6$

(b) Find the locus of the complex number $|Z - 1 - i| < 3$, represent it on the Argand diagram and shade the region representing it.

(c) (i) Solve the equations $Z^3 - 8i = 0$ and $z^3+i=0$.

(ii) Find in the form $a + ib$, the roots of the equation $Z^3 - 26 - 18i = 0$

(d) If z is a complex number,describe and illustrate on the argand diagram this locus

Given by each; (i) $\left|\frac{z-2i}{z-i}\right|$=2 (ii) $\arg(z + 2 + 3i) = \frac{\pi}{3}$.

(e) Show that the locus of the complex number Z of $|z+1|=2|z-1|$ is an ellipse and Find its equation,hence find its focus,eccentricity and equation of directrix.

10 Solve the following by row reduction to echelon form.

(b) $3x-2y+4z=-7$,$x+y-6z=-10$ and $2x+3y+2z=3$

(c) By using the method of synthetic division, divide

$6x^4-17x^3+22x-9$ by 2x -3

(d) Find the equation of circle that touches line y=x at pt (3,3) and passes thru B(5,9).

(e) Solve the simultaneous equations

$2^x+4^y=12$ and $3(2^x)-2(2^{2y})=16$ .Hence show that $4^x+4(3^{2y})=100$

(f) When the quadratic expression ap^2+ bp + c is divided by p-1, p-2 and p+1, the remainders are 1, 1 and 25 respectively. Determine the factors of the expression.

11. (a) Find the term independent of x in the expansion of $(2x-\frac{1}{x^2})^{12}$

(b) Expand $(\frac{1+x}{1+3x})^{1/3}$ up to the term. Hence by taking x = $1/125$, use your result to calculate the cube root of 63 correct to four decimal places.

12. (a) Show that $\cos^{-1}\left(\frac{63}{65}\right)+2\tan^{-1}\left(\frac{1}{5}\right)=\sin^{-1}\left(\frac{3}{5}\right)$

(b) Solve the equation $(i) Sin\, 3x+{}^{1}/_{2}=2\, Cos^{2}$ x for 0°≤ x ≤ 360°

(ii) 4sinxcos2xsin3x=1 for 0° ≤x≤180°

(c) Show that in any triangle A B C, $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$. Then solve the triangle two sides 5 and 7 and the included angle of 45°.

(d) Show that for allvalues of $\theta, cos\theta+\cos\left(\theta+\frac{2}{3}\pi\right)+\cos\left(\theta+\frac{4}{3}\pi\right)=0.$

Hence show that $cos^2\theta+cos^2\left(\theta+\frac{2}{3}\pi\right)+c0s^2\left(\theta+\frac{4}{3}\pi\right)=\frac{3}{2}.$

(e) (i) Prove that $4\cos\theta 3\cos\theta+1=\frac{\sin 5\theta}{sin\theta}$

(ii) In any triangle ABC,Prove that TanBcotC= $\frac{a^2+b^2-c^2}{a^2-b^2+c^2}$.

13. (a) Solve the inequality,

(i) $\frac{x-1}{x-2}>\frac{x-2}{x+3}$ (ii) $2|x-1|=|x+3|$ (iii) $|5x-6|=x^2$

(b) Sketch the curve $f(x) = \frac{x^2+2x+3}{x^2+3x+2}$.

(c) If $y = \frac{(x-1)^2}{(x+1)(x-3)}$. Find the values of y for which the curve is not defined,

Hence find the nature of the turning points. sketch the curve.

(d) Find the co-ordinate of intersection of $y = \frac{x}{x^2+1}$ and $y = \frac{x}{x+3}$.

Sketch both curves on Same axes and show that the area of finite region in the first quadrant enclosed by two curves is $\frac{7}{2} \ln 5$ -3ln3-2.

(e) The domain of the function defined by $f(x) = \frac{4(2x-7)}{(x-3)(x+1)}$ is the set of all real

Values of x other than 3 and -1.Express f(x) in partial fractions,hence or

Otherwise,show that $f^1(x)$ is zero for two +ve integral values of x.Find the

Turning points of this function.Hence sketch f(x).Dertermine the area of the

Region bounded by the cyrve,the x-axis and the lines x=4,x=6.

(Leave your answer in logarithmic form.)

14. (a) Show that at any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ may be written as (a sec θ, b tan θ)

(b) Find c in terms of a, b , m if y = m x + c is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .Find the asymptotes.

(c) The normal at the point P(5 Cosθ,4sinθ) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the

X and Y axes at Land M respectively. Show that the locus of R ,the midpoint of

LM is an ellipse having the same eccentricity as the given ellipse.

(d) The curve $b^2x^2 + a^2y^2 = a^2b^2$intersects the positive X-axis at

A and the Y-axis at B.

(i) Determine the equation of the perpendicular bisector of AB.

(ii) Given that this line intersects the X-axis at P and that M is the bisection of AB.Show that the area of triangle PMA is $\frac{b(a^2+b^2)}{8a}$

(e) (i) State the vertex,focus and directix of **$8y^2 + 6y - 9 = 4x$** and **$2x^2 + 3y^2 - 4x + 9y + 5 = 0$.**

(ii) Show that the two tangents of gradients m to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are

$Y = mx \pm \sqrt{(a^2m^2 + b^2)}$.find eqnsof tangents to ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ at (-2,5).

15. (a) Find the angle between $\frac{x-3}{2} = \frac{2y-1}{4} = \frac{3z-1}{4}$ and $-4x + 3y + 2z = 7$

(b) Find the angle between the line of intersection of the planes 2x + y + 3z = 4 and 3x + 2y + 2z = 7 and the line $\frac{1-x}{1} = \frac{y-2}{2} = \frac{z-3}{4}$

(c) Find the distance between the planes 2x – 3y + 4z = 7 and 8x – 12y + 16z = 6.

(d) Find the equation of the plane through the (1,0,-1) and containing the line $X = -y = \frac{z}{2}$.

(e) Find the equation of the plane containing the lines $\frac{x-3}{5} = \frac{y+1}{2} = \frac{z-3}{1}$ *and* $\frac{3-x}{-2} = \frac{y+1}{4} = \frac{z-3}{3}$.

(f) Find the equation of the plane through the point (1,-2,1) which also Contains the line of intersection of the planes x+y+z+6=0 and x-y+z+5=0.

(g) Show that vectors $i - 2k, -2i + j + 3k$ and –i+j+k form a triangle . Hence or otherwise find the area of the triangle.

(h) Show that the vectors **a=3i+j-4k,b=4i-j-3k,c=5i-3j-2k** are coplanar.

(I) The equations $\frac{x-3}{2} = \frac{y-5}{1} = \frac{z-2}{-3}$ *and* $\frac{x+1}{3} = \frac{y+4}{1} = \frac{z-2}{-2}$ represent pipes A and B in a chemical plant. Find the the length of the shortest pipe that can be fitted at its end points.

- ***TO THE PROBLEMS OF YOUR LIFE,YOU'RE THE SOLUTION ,AND TO THE THE QUESTIONS OF YOUR LIFE ,YOU'RE THE SOLUTION.***
- ***IF YOU ARE GOING TO ACHIEVE EXCELLENCE IN BIG THINGS, YOU DEVELOP THE HABIT IN LITTLE MATTERS.***
- ***WHEN SPIDER WEBS UNITE, THEY CAN TIE UP A LION.***
- ***PRAYER AND HARD WORK GO TOGETHER WITH DISCPLINE.***
- ***THINGS DO NOT CHANGE; WE CHANGE***

GOOD LUCK AND SUCCES

”””” **He who thinks that he can make it,makes it””**...