

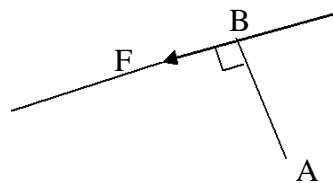
MOMENTS

1. A door is easily closed by pushing on the edge further from the hinge than by pushing at a point near the hinge.
2. A child on one end of a seesaw can balance a heavier adult near the center of the seesaw.

In each of the examples above, the application of a force is causing a body to rotate about an axis. This turning effect is called the moment of the force.

DEFINITION

Moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

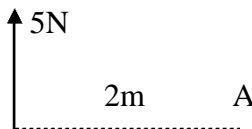


$$\text{Moment about A} = F \times AB$$

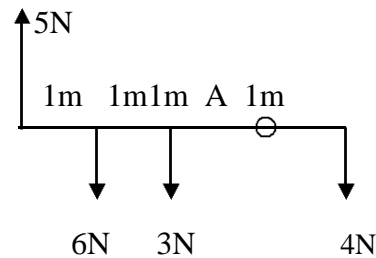
EXERCISE

1. Find the moment about the point A of the forces shown in each diagram.

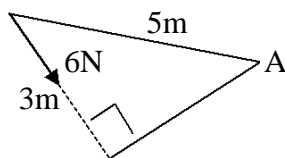
(i)



(ii)



(iii)



Ans. (i) **10Nm clockwise**, (ii) **4Nm clockwise**, (iii) **24Nm anti-clockwise**.

2. A force of $(3\mathbf{i} - 2\mathbf{j})\text{N}$ acts at the point which has position vector $(5\mathbf{i} + \mathbf{j})\text{m}$. Find the moment of this force about the point which has position vector $(\mathbf{i} + 2\mathbf{j})$. Ans. **5Nm clockwise**.
3. A force of $(3\mathbf{i} + 2\mathbf{j})\text{n}$ acts at the point which has position vector $(5\mathbf{i} + \mathbf{j})\text{m}$ and a force of $(\mathbf{i} + \mathbf{j})\text{N}$ acts at the point which has position vector $(2\mathbf{i} + \mathbf{j})\text{m}$. Find the sum of the moments of these forces about the point which has position vector $(\mathbf{i} + 3\mathbf{j})\text{m}$. Ans. **17Nm ant-clockwise**.

LIKE AND UNLIKE FORCES

Like forces are forces which are parallel and act in the same direction, while unlike forces are forces, which are parallel but act in opposite directions.

COUPLE

Two unlike forces of equal magnitude, not acting along the same line, form a couple. A couple has a moment, but cannot cause translation.

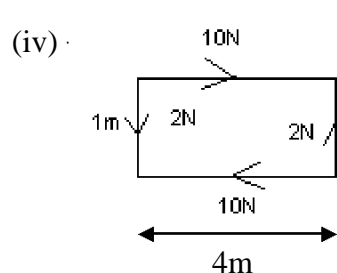
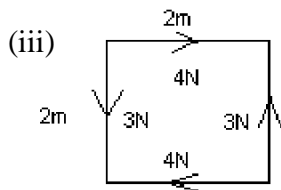
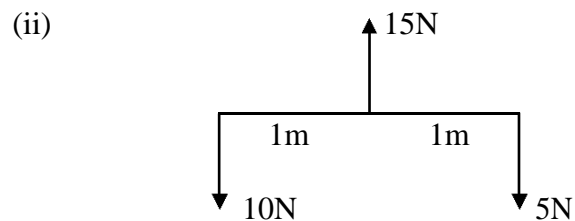
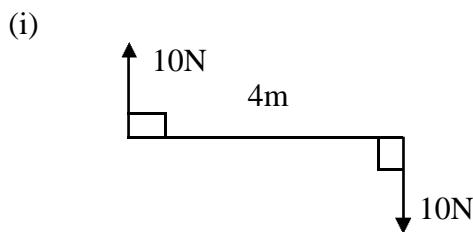
MOMENT OF A COUPLE

Let the magnitude of each force forming a couple be F N, and the perpendicular distance between their lines of action be x m. Moment about any point = $F \times x$ Nm.

The turning effect of a couple is independent of the point about which the turning is taking place.

EXERCISE

1. Find which of the following systems will reduce to a couple and in these cases, find the moment of the couple.



(i) **40Nm clockwise**, (ii) **5Nm ant-clockwise**, (iii) **2Nm clockwise**, (iv) **2Nm clockwise**.

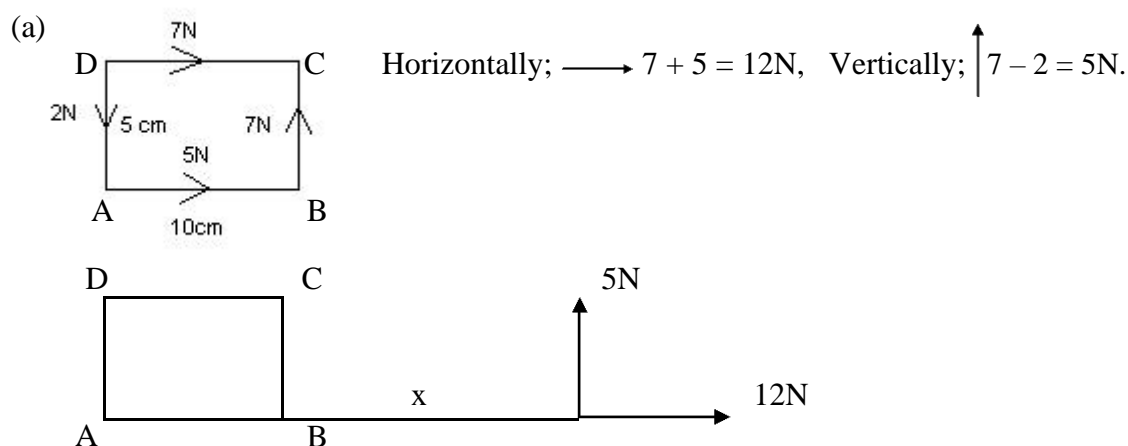
2. ABCD is a rectangle with $AB = 5\text{m}$ and $BC = 2\text{m}$. A force of 3N acts along each of the four sides AB, BC, CD and DA in the directions indicated by the order of the letters. Show that the forces form a couple and find its moment. Ans. **21Nm in sense ABCD**.
3. ABCD is a square of side 40cm . Forces of 20N , 15N and 20N act along the sides AB, BC and CD respectively and a force Y acts along DA. The directions of the forces are given by the order of the letters. If the system is equivalent to a couple, find the magnitude of Y and the moment of the couple. Ans. **15N, 14Nm in the sense ABCD**.

4. Forces of $(ai + bj)N$ and $(6i - 4j)N$ act at the points having position vectors $(-2i - 2j)m$ and $(3i - j)m$ respectively. If these forces reduce to a couple, find a and b and the moment of the couple. **Ans. $-6, 4, 26Nm$ clockwise.**

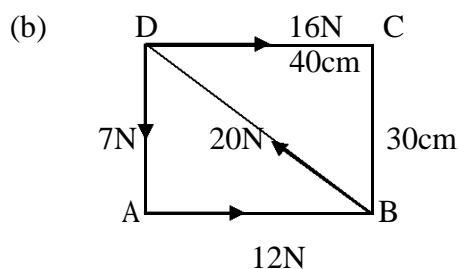
EQUIVALENT SYSTEM OF FORCES

Any system of coplanar forces is either equivalent to a couple or can be replaced by a single force or resultant. The direction of the resultant is found by resolving and taking moments.

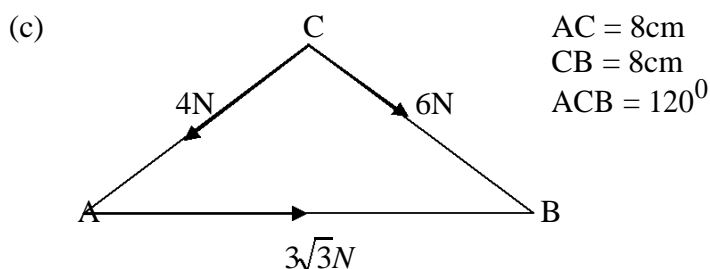
1. Find the magnitude and direction of the single force equivalent to the system of forces shown. Find also where the line of action of this single force crosses AB .



Moments about A ; $5x = 7 \times 10 + -7 \times 5$. Therefore $x = 7$ cm. **Ans. $13N, 22.6^\circ$ to AB . 7 cm from A.**



(b) **$13N, 22.6^\circ$ to AB , at A,**



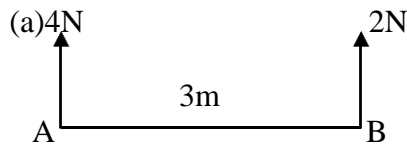
(c) **$8.54N, 324.2^\circ$ to AB , $8.31cm$ from A**

2. ABCD is a rectangle with $AB = 1.5m$ and $AD = 1m$. Forces of $2N, 1N, 1N$ and $3N$ act along AB, BC, Dc and AD respectively. Calculate the magnitude and direction of the single force that could replace the system of forces and find where its line of action cuts AB . **Ans. $5N, 53.1^\circ$ to AB , $12.5cm$ from A.**
3. ABC is an isosceles triangle, right-angled at A with $AB = 1m$. Forces of $8N, 4N$ and $6N$ act along BA, BC and CA respectively. Find the single force that could replace this system and find where its line of action cuts AB . **Ans. $11.3N, 16.3^\circ$ to BA . $89cm$ from A on BA produced.**
4. Point O is the origin and points A, B, C and D have position vectors $(3i + j)m, (i + 3j)m, (-2i + j)m$ and $(-2i - 2j)m$ respectively. Forces of $(3i + 3j)N, (4i - 5j)N, (-5i + 2j)N$ and $(2i + 3j)N$ act at points A, B, C and D respectively. Find the single force that could replace this system and find where its

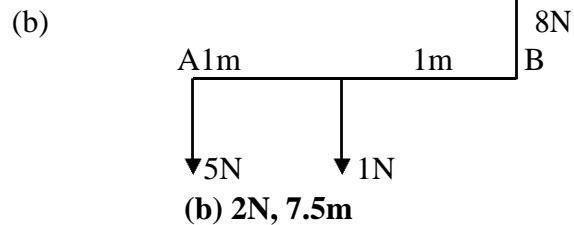
line of action cuts the horizontal axis through O. A couple of moment a Nm anti-clockwise and a force $(bi + cj)$ N acting through the point which has position vector $(2i + j)$ m are now added to the system. If these reduce the system to equilibrium, find a, b and c.

Ans. $(4i + 3j)$ N, $-4i$, $a = 14$, $b = -4$, $c = -3$.

5. In each of the following cases, find the magnitude of the resultant of the forces shown and the distance of its line of action from A.

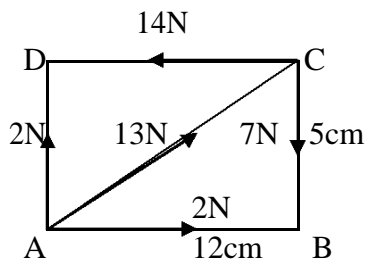


(a) **6N, 1m**



(b) **2N, 7.5m**

6. Two like vertical forces of 4N and QN act at points A and B respectively where AB is horizontal and of length 6m. Their resultant is a force of PN and acts at X, between A and B, where AX = 2m. Find P and Q. Ans. **6, 2**.
7. Show that the following system of forces is, equivalent to a couple and find the moment of the couple.



Ans. **0.14Nm in sense ADCB**

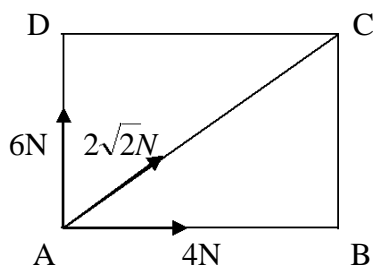
8. ABCD is a rectangle with AB = 4m and BC = 3m. Forces of 4N, 5N and 10N act along CB, DC and DB respectively. Find the single force equivalent to this system and find where its line of action cuts AB. A couple of moment 15Nm, in the sense ABCD, is now introduced to the system. Find the magnitude and direction of the single force that will replace this new system and show that its line of action passes through B.

Ans. **16.4N, 37.6° to AB, 5.5 m from A on AB produced. Same force after couple introduced.**

REVISION QUESTIONS

1. The forces 3N, 4N, 5N and 6N act along the sides AB, BC, CD and DA of a rectangle. Their directions are in the order of the letters. \overline{BC} is horizontal. Find the resultant force and the couple at the centre of the rectangle of sides 22m by 4m.

2.



Three forces act along AD, AB and AC as shown above. ABCD is square. Determine the resultant of the three forces and the angle which it makes with AB.

3. AB is a uniform rod of weight 10N and length 8m. It is smoothly hinged at A and is kept in equilibrium in a horizontal position by two inelastic strings BC and DC. Given that $AC = AD = 6\text{m}$ and the tensions in CD and CB are $T\sqrt{2}$ and T respectively. Calculate the;
- value of T
 - reaction at A
4. Forces of 6N, 6N, 4N and 8N act along AB, CB, CD and AD respectively in the directions indicated by the order of the letters of a square ABCD of side 4m. Find the;
- magnitude of the resultant force
 - equation of the line of action of the resultant if it cuts BA produced.
5. The centre of a regular hexagon ABCDEF of side $2a$ metres is O. Forces of magnitude 4N, s N, t N, 1N, 7N and 3N act along the sides AB, BC, CD, DE, EF and FA respectively. Their directions are in the order of the letters.
- Given that the resultant of these six forces is of magnitude $2\sqrt{3}$ N acting in a direction perpendicular to BC, determine the values of s and t . Ans:
 - (i) Show that the sum of the moment of the forces about O is $27a\sqrt{3}$ Nm.
(ii) If the mid-point of BC is M, find the equation of the line of action of the resultant, refer to OM as x-axis and OD as y-axis.
6. Four forces represented by $i - 4j$, $3i + 6j$, $-9i + j$ and $5i - 3j$, at a points with position vectors $3i - j$, $2i + 2j$, $-i - j$ and $-3i + 4j$ respectively.
- Show that the forces reduce to a couple and find its moment
 - If the fourth force is removed, find the magnitude and direction of the resultant forces.
7. Coplanar forces $F_1 = i - 2j$, $F_2 = 3i + 4j$ and $F_3 = -5i + j$ Newtons act at points with position vectors $4i + 5j$, $3i - 2j$ and $i + 4j$ respectively. Find the Cartesian equation of the line of action of the resultant force.
8. The forces $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$, $\begin{pmatrix} -10 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 7 \\ -7 \end{pmatrix}$, $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ N act at the points (2,2), (5,0), (-4,4), (0,-5) and (6,0) respectively.
- Show that the system reduces to a couple and state the moment of the couple.
 - The third force is removed and a couple added to the new system. Given that the line of action of the resultant of the forces cuts the y - axis at 12 units, find the couple.

9. A uniform ladder **AB** of weight **3N** stands on a rough horizontal surface at **A** and leans against a smooth rigid horizontal peg at point **C**, where $AC = \frac{2}{3} AB$. Given that the ladder is in limiting equilibrium making an angle of 30° with the surface.
- Find the (i) normal reactions at **A** and **C**
(ii) coefficient of friction at **A**
 - If a boy of weight **W** starts to climb the ladder, determine whether it slides before toppling.
10. Forces of magnitudes **6N**, **6N**, **4N**, **10N** and **8N** act along **AB**, **BC**, **CD**, **DB** and **AD** respectively, in the directions indicated by the order of the letters of a rectangle **ABCD** of dimensions 4m by 3m. Find the;
- magnitude of their resultant
 - equation of the line of action of their resultant force
 - distance from **B** where it cuts **AB**